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Modeling a thermal power plant drum-type boiler for control: a parameter identification approach

Chin Chen
Iowa State University

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Modeling a thermal power plant

drum-type boiler for control:

A parameter identification approach

by

Chin Chen

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I. INTRODUCTION

A. General Statement

The principal functions of a power system are to convert energy from various forms to electric energy and to transmit this energy to consumers in diversified areas. The smooth flow of energy to all parts of a power system is a fundamental requirement. To satisfy this requirement, it is desirable that the power generation units be properly controlled so that the production and consumption of energy can be maintained in equilibrium at all times.

At the present time, the majority of the electric power generation units are thermal power plants. Although these plants are subject to frequent adjustments in response to load variations, the control of thermal power plants has not always been adequate (4); the control of the power plant boilers is generally accomplished by a number of independent analog or direct digital control (DDC) devices designed on a single-input and single-output basis. When an error is detected from a certain variable, the corresponding control device starts to act. This will cause error to other variables, and the other related control devices start to act. The control devices will adjust to each other until the disturbances subside. This kind of control is very slow and ineffective. For a multiple-input and multiple-output system, it is desirable to have multivariable control to produce fast and effective control action under specified criteria.

In this research a model for a thermal power plant boiler of forced circulation is developed. The model is intended for use in multivariable control studies on boilers for the following purposes:

a) for automated control of steam power generation, and

b) to minimize fluctuations in boiler pressure and temperature caused by load changes and control actions.

B. Modeling Approaches for a Boiler

A boiler unit is a distributed system involving mainly fluid flow and heat transfer processes. The fluid flow path of the boiler in connection with the turbine is shown in Fig. 1. The field equations describing these physical processes are generally in the form of nonlinear partial differential equations. Because of the complexity in boiler geometries and existence of heat capacitances which cause thermal delays, the physical phenomena in a boiler are further complicated. It is very difficult to represent the field equations for boiler system dynamics and to obtain solutions or to simulate them on computers. Some alternative modeling approaches have been considered by different boiler model investigators.

1. Physical approach

One method is to make some simplifying assumptions on the physical processes so that the process can be described by lumped equations. The heat transfer processes are represented by empirical equations. The equation coefficients are then determined from the physical data of the boiler.

The resultant model equations will include nonlinear differential equations and nonlinear algebraic equations. Since it is very difficult to design control systems for nonlinear systems, these equations are usually linearized with respect to steady-state operating points. After the mathematical linearization the boiler dynamics are then represented by a
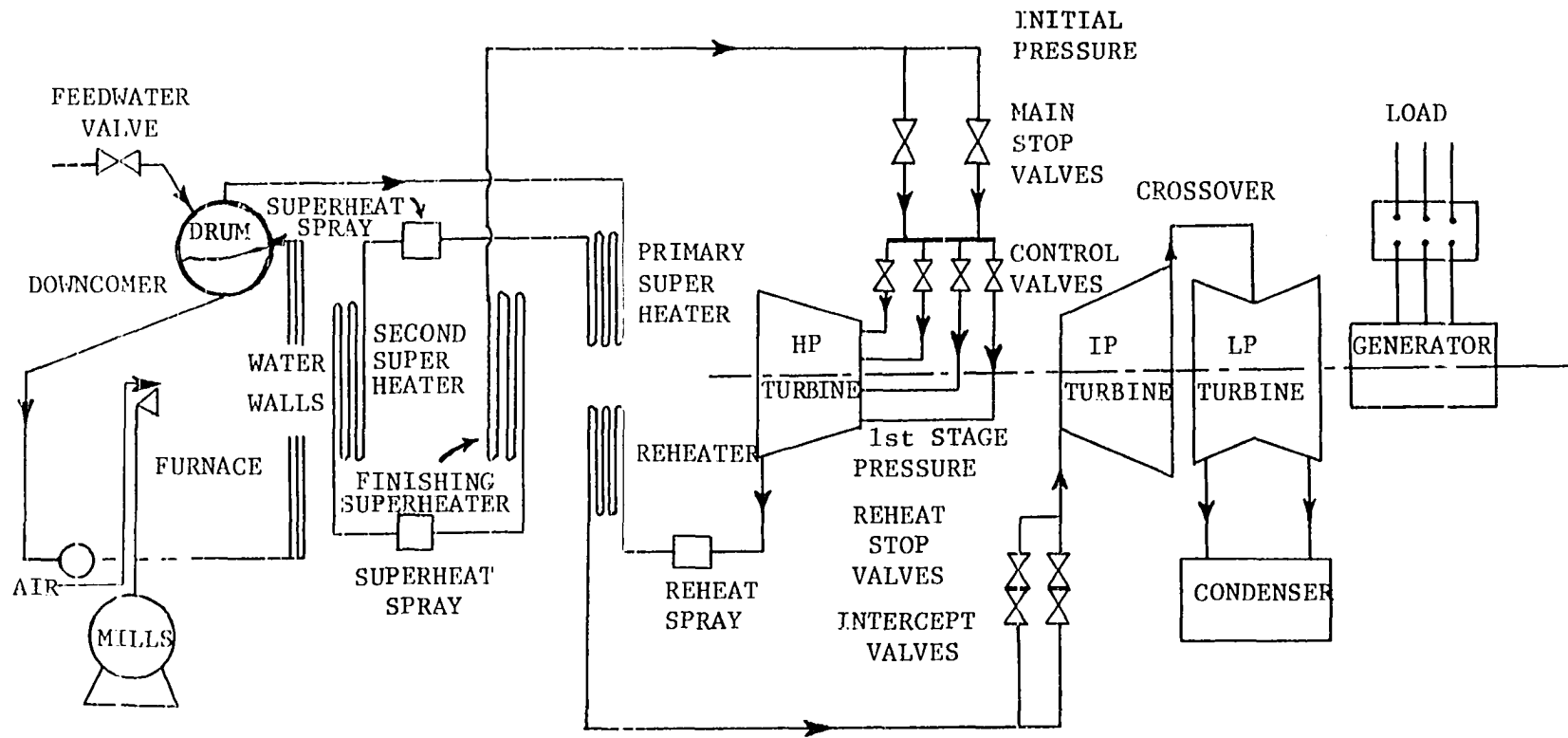
Fig. 1. Fluid-flow path for a typical drum-type boiler and single reheat turbine.

set of linear differential equations and linear algebraic equations. The model constants are generally computed from boiler design data and experimentally tested boiler operating constants.

Chien, Ergin, Ling, and Lee (5) studied the dynamics of a boiler and described analytically the development of a linear dynamic model for a boiler with this approach. This paper is the first publication which gives a comprehensive analysis of a boiler system. The boiler considered was a drum-type, oil fired naval unit with natural circulation. The boiler was divided into four sections in the analysis:

1) superheater,

2) downcomer-riser loop,

3) drum, and

4) gas path.

The important simplifying assumptions made on the boiler processes in the development of the model equations are the following (5):

a) vapor and liquid velocities in the upriser are identical,

b) heat-transfer rate to boiling liquid from the waterwall tube is proportional to the cube of the temperature difference between the wall and the liquid-vapor mixture (empirical equation),

c) quality is constant throughout the upriser,

d) temperature of liquid-vapor mixture in the upriser is always the same as the saturation temperature corresponding to drum pressure,

e) downcomer liquid temperature is the same as the drum liquid temperature,

f) there is no temperature gradient in the vapor phase in the drum and the temperature is always the saturation temperature corresponding to drum pressure,

g) liquid phase in the drum has no temperature gradient except through a very thin layer at the liquid surface,

h) evaporation or condensation rate in the drum is proportional to the difference of liquid and saturation temperatures,

i) liquid-level changes due to bubble formation in the drum are neglected,

j) the effect of the economizer on the overall system dynamics is neglected, and the feedwater temperature is assumed constant,

k) the air-fuel ratio is assumed constant,

l) in each tube bank the heat-transfer rate is determined by the tube-wall temperature and the average gas temperature (empirical equation),

m) inertia of the hot gas is neglected, and

n) delays of gas temperature changes due to the heat capacitance of the hot gas are neglected.

The boiler model given in this paper was considered too simple and not accurate (19). However, the analysis and modeling approach has been followed by many later boiler model investigators. In efforts for improvements of model accuracy, variations exist in the following areas for the boiler models proposed by different investigators:

a) the simplifying assumptions on physical processes,

b) the choice of empirical heat transfer equations,

c) the definition of model variables, and

d) the method of obtaining model coefficients.

It should be noticed that the following factors have been considered most significant causes of inaccuracy in a linear boiler model developed with this kind of approach:

a) Empirical heat transfer equations represent only the overall effect of heat transfer but do not explain the actual mechanism; these equations may not be accurate when the temperature variation through the heat transfer section is large.

b) The simplifying assumptions for the boiler processes may be inadequate in describing the processes.

c) As temperature distributions on heat transfer surfaces are not uniform, there are problems of how to obtain accurate model coefficients.

d) The incremental equations are accurate only for the steady-states or their neighborhood where the model coefficients are calculated. In practical boiler operation, it is difficult to maintain the boiler variables at steady-state values for a longer period when there is no adequate multivariable control on the boiler. Therefore, the boiler may not always operate under the conditions where the model constants are calculated.

e) The steady-state operating point constants of a boiler are difficult to measure for the same reason mentioned in (d). The error on the measurement can be large.

Some other linear boiler models have been given by Daniels, Enns and
Hottenstine (7), and Kwan and Anderson (15). The boiler considered by
Daniels was the same one considered by Thompson at a later time. It was a
drum-type, coal-fired unit with forced circulation. The use of the average
of end point values of variables such as temperature, flow rates, density
for each heat transfer section in this model can cause erroneous transient
response as explained earlier by Thal-Larsen (29). The boiler considered by
Kwan and Anderson was a drum-type coal-fired unit with natural circulation.
The dynamics of the downcomer and economizer were included in the model.
The mass balance equation, energy balance equation, momentum equation,
and metal heat balance equation were given for each transfer section. The
model was represented by 107 algebraic and differential equations. The
model equations used by Kwan, Daniels, and Chien are similar, but the
definitions of the variables are different. Chien defined the variables for
each heat transfer section to be the average values of the sections, and
Kwan defined them to be the values at the outlet of the sections. None of
these models was treated correctly in view of the inaccurate factors men-
tioned before.

In order to obtain more accurate model coefficients so as to improve
model accuracy, it has been suggested to divide each heat transfer section
into small "elemental sections." Accurate steady-state thermal properties,
such as temperature, pressure, and enthalpy, of hot gas, water, steam, or
water-steam mixture are then obtained for all the boundary points of ele-
mental sections. Since the differences of variable values between two
nearby boundary points is small, the model equation coefficients and the
thermal-physical properties along the heat transfer path can be determined

more accurately. The details of using this kind of technique have been explained by Thompson (30,31) and Shang (27). Thompson developed a linear model for a drum-type utility boiler, and Shang developed linear models for once-through boilers. In both cases experimental data measured from heat transfer sections for a steady-state operation level of the respective boiler were available for the determination of accurate steady-state thermal profiles. Extensive numerical computations were made in determining steady-state profiles and coefficient values with a digital computer.

The accuracy of the results is dependent upon the number of elemental sections into which a heat transfer section is divided. However, when more elemental sections are used, the number of model variables increases accordingly. The boiler model then becomes complex. To avoid having a boiler model too large in size, Shang (27) also suggested combining some elemental sections to form lumps along the heat transfer path. Fewer lumps can be used for heat transfer sections with less important storage behavior, such as the economizer, and more lumps should be used in the sections with important storage behavior, such as the superheater.

The data provided by Thompson in the comparison of experimental results of the boiler with responses of the model, where each heat transfer section was divided into five elemental sections, showed that the model responses were more accurate than those given by Daniels, but the accuracy was still not satisfactory. The data provided by Shang showed better accuracy of model responses. However, the limited comparison data available were not enough to justify the model accuracy. In both cases, the models do not satisfy the requirement of simplicity for control design purposes.

Several persons, including Kwatny et al. (16), McDonald (18),
McDonald and Kwatny (19), and McDonald et al. (20), have studied nonlinear
models for a boiler-turbine unit, expecting that a nonlinear model could
cover a wider range of boiler loads. The boiler was the same one con-
sidered by Daniel et al. (7) and Thompson (30). Since a nonlinear model is
complex itself, care was taken to keep the model in its simplest possible
form. The model given by Kwatny, McDonald, and Spare (16) was actually the
same model given by McDonald and Kwatny (19). Some process equations used
in the model do not adequately represent the actual processes, especially
those representing the transfer of heat through the waterwall tubes and
into the drum. There is a major heat transfer delay in the transfer of
heat through the tube walls and into the drum system, and the effect of
these processes on the dynamic performance of the boiler is significant.
McDonald (18) proposed a nonlinear boiler model at a later time where the
effect of tube wall metal on heat transfer was considered, but the model
was still not well-defined.

There are many difficulties involved in the determination of a non-
linear model to be valid for wider boiler operation ranges. The obvious
ones among them are the following:

a) The empirical heat transfer equations may not be valid for a wide
   range of state variations because these equations are determined
   from the observations of steady-state heat transfer.

b) Model coefficients become functions of the boiler states, and
   relations among them are hard to determine.

2. Black-box approach

Another method of modeling a physical system is called the "black-box method." The dynamic performance of a physical system is observed from its output response with respect to several input signals. Then the possible model transfer functions which match the input-output relations are investigated. The determination of model transfer functions generally requires intuition, knowledge in systems theory, and experience with the thermal system. The feature of modeling with this approach is that only the overall system behavior is required for the model, and it usually leads to simple mathematical form.

Since the model is developed based on the dynamic response of the system, it can describe the system dynamics very well if the model is correct. However, since the transfer function which can fit one set of input-output data is not unique, frequently a model for a complex multivariable system is derived which accurately fits one set of input-output data but is inaccurate for a different set. This is particularly true for systems involving nonlinear processes.

Some black-box models for thermal power plant boilers have been proposed, e.g. by Profos (23), de Mello and Imad(8), Laubli and Fenton (17) and de Mello, Mills and B'rells (9). They have been included and discussed in a recent publication by Anderson (1).

One common problem with a black-box model is how to find the model constants. In earlier days, it was suggested to simulate the model on an analog computer and to find the model constants by tuning the potentiometers on the analog computer and comparing the response of the model with

the response of the physical system for step inputs or ramp inputs. This

approach is time consuming and difficult to obtain accurate results,

especially for a multivariable system. With the progress in system identi-

fication, the parameter identification technique becomes available for pro-

viding a convenient means of estimating model constants. The input-output

data measured from the physical system with inputs perturbed are required

for the identification computation. The computations are performed on a

digital computer. The input-output data should be measured from the test

on the physical system with inputs perturbed. This kind of measurement is

usually easier to do than measuring the steady-state operation point values

as required for computations with models developed with the physical

approach.

Applications of the parameter identification technique to identify

parameters for some thermal and nuclear power plant models have been

reported (10,12,21,22,24,25). Among them Park (22) and Eklund and

Gustavsson (10) identified thermal power plant boiler models. The idenci-

fication by Eklund and Gustavsson was based on single input experiments,

and that by Park was based on multiple input experiments. The boiler model

identified by Park was very close to that proposed by Laubli and Fenton (17).

The model inputs are fuel flow rate and control valve position, and the

model outputs are throttle pressure and steam flow rate. The measured

dynamic response data from a utility fossil power plant boiler were used to

compute the model parameter constants. The results showed that the outputs

of a model with constants computed with a set of dynamic data agree very

well with the measured outputs for a period of about twenty to thirty

minutes. However, the model with parameters computed with one set of data could not produce comparable output responses for a different set of physical data measured at nearly the same power level.

3. The modeling approach in this research

Because of the shortcomings of the boiler models developed with the previous two approaches, in which a model developed with physical approach is too complex for control studies and a model developed with black-box method may not represent the boiler for a longer period, a boiler model developed here will utilize the advantages available from these two approaches. The resultant model may be considered as a more accurate black-box model.

The model equations will be developed based on physical principles, and the advantageous parameter identification method can be used to compute the model constants. With model constants computed by this method, it is no longer necessary to divide the heat transfer section into elemental sections. Thus the resultant model will have the complexity at most that of the simplest physical model. Since the model constants can be computed with the dynamic response data of the boiler, the model will be able to describe the boiler dynamics very well.

In order to obtain an accurate model suitable for control studies, all the system processes which have significant effects to the overall system behavior will be included. Unlike all the black-box models given before in which the temperature features of a boiler are not included, the model developed here will include important pressure features as well as

temperature features. Temperature features are important for boiler controls, as will be explained later.

The model variables, such as temperature, pressure, and mass flow rate for each boiler component will be defined to be the "effective average values" of the corresponding properties. The physical equations which are nonlinear will be linearized with respect to steady-state operation points. Although this will result in a model which is accurate only around these operation points, the model can be accurate for a wide load range of the boiler when the temperatures across the heat transfer sections and the drum pressure are maintained to have minimum deviation from the desired operation points during load changes with suitable multivariable control. This scheme is possible because the rate of heat transfer is a function of temperature gradient and medium mass flow rate. When the temperature gradient is fixed, the rate of heat transfer becomes a function of mass flow rate. The heat transfer equations then become closer to linear and valid for a wider range of linear perturbations.

Care should be taken in measuring the dynamic physical data for identification computation. The boiler must operate under desired steady-state operation conditions for a period before the inputs are perturbed and the data are recorded.

II. BOILER PROCESSES

The boiler is constructed mainly of metal tubes and a drum. The burning of fuel produces heat in the furnace which is essentially surrounded by waterwall tubes. The hot gas is drafted by fans. Waterwall tubes, superheater tubes, reheater tubes, economizer, and air heater are located in a gas passage and absorb heat from the hot gas. The fraction of heat which is not absorbed is lost to the air through the stack. Figure 2 shows the diagram of the gas flow path.

Inside the different boiler sections, flows water, steam, or a mixture of both. The saturated water at drum pressure enters the downcomer at the downcomer inlets located at the bottom of the drum. While circulating through the waterwall tubes, the water absorbs heat and is converted partially into steam. The mixture of steam and water discharges into the drum at the waterwall outlets located at the upper portion of the drum, where the water and the steam are separated. When part of the water in the drum system is converted into steam, the water level in the drum decreases at the same time. A feedwater control is used to feed water out of the economizer into the drum so that the water level in the drum can be maintained. The steam in the drum leaves the drum and passes through superheaters before going through the throttle valves. The steam gains additional energy in the superheaters and becomes superheated at the throttle valve. The steam with high heat energy potential is then discharged through the throttle valve into the turbine in which part of the heat energy is converted into mechanical energy that drives the turbine. The steam and water flow paths are shown in Fig. 3.
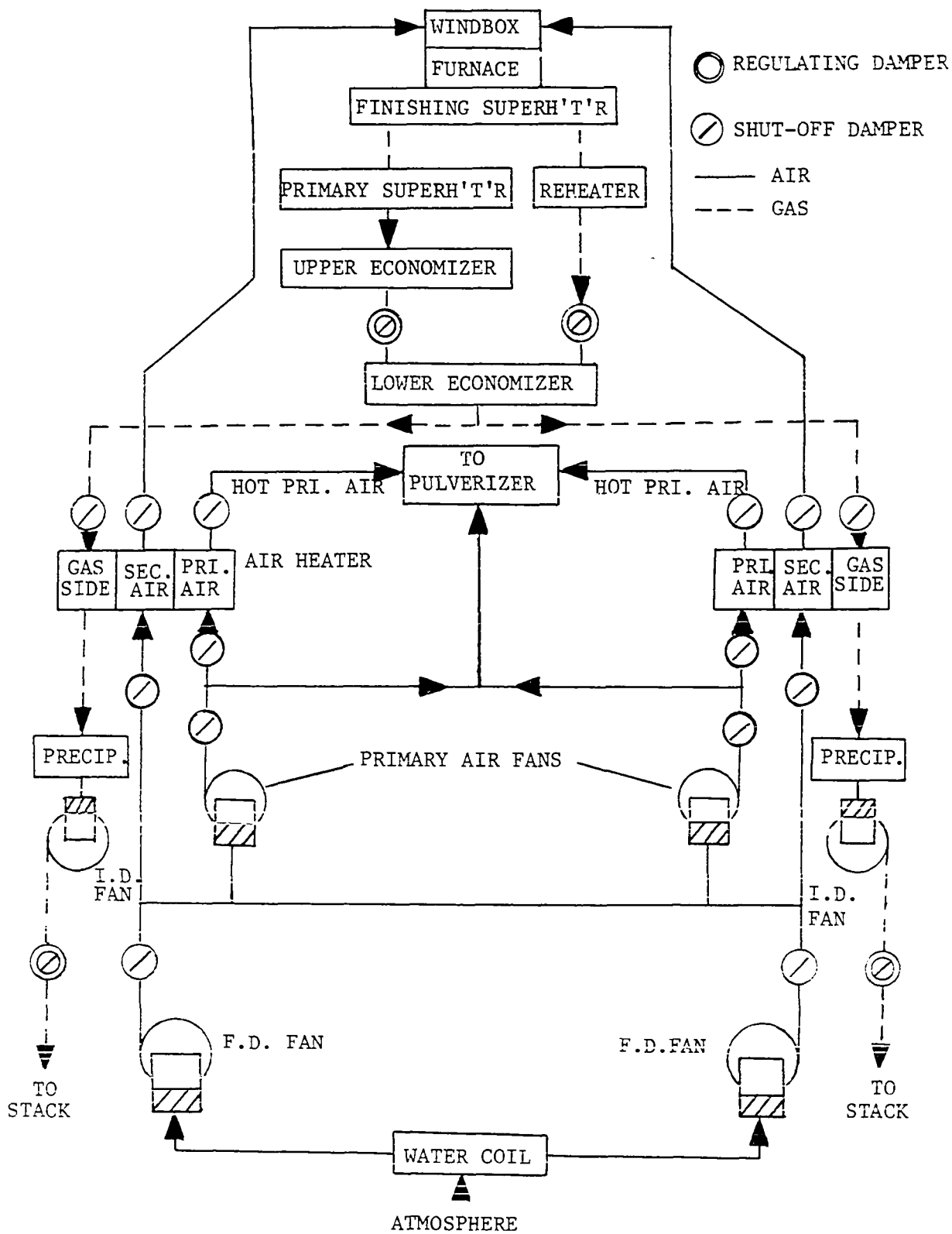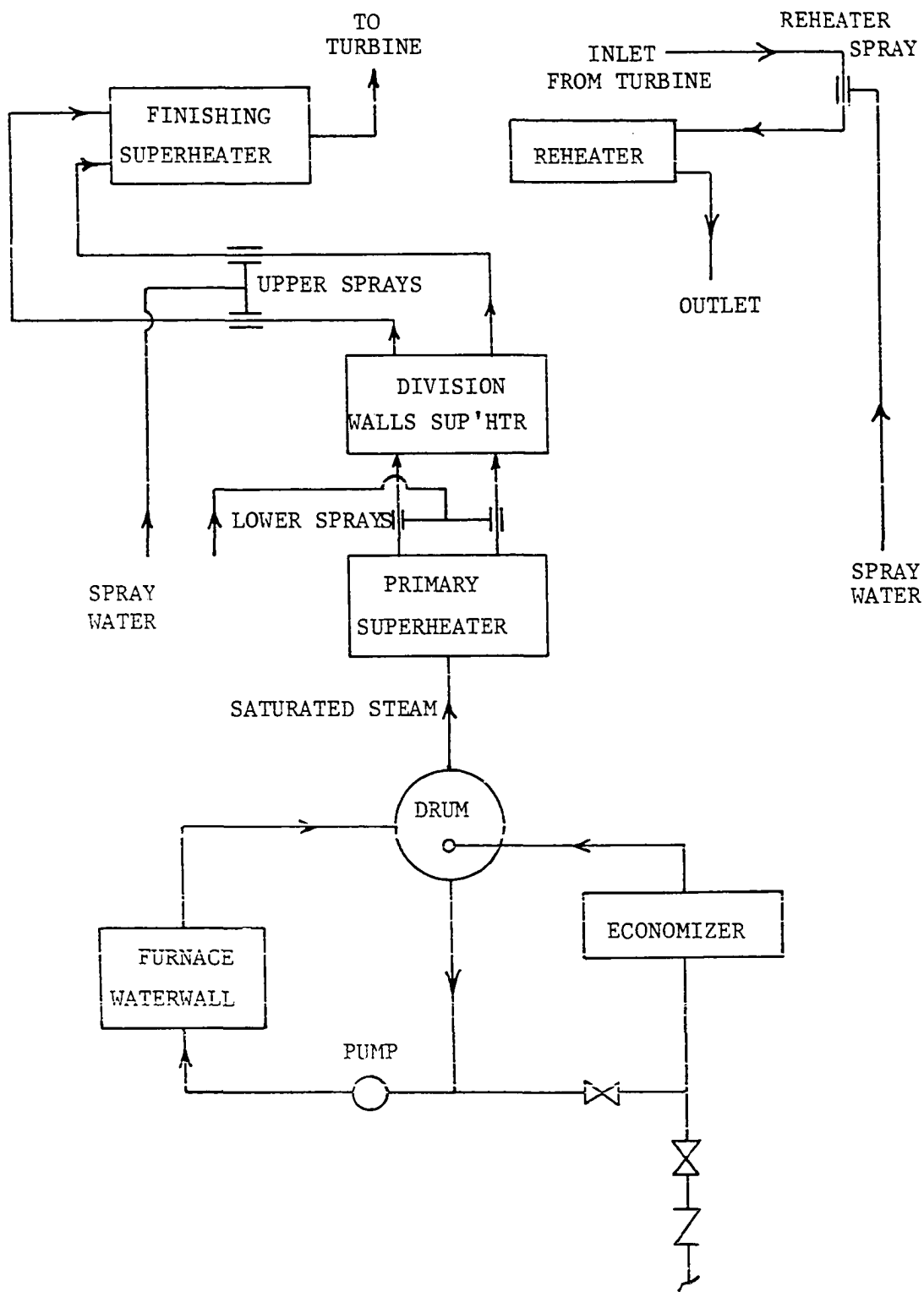
Fig. 2. Air and gas flow.

Fig. 3. Steam and water flow.

In the model development, the "effective average" values of process variables are used to describe the processes they represent. Thus the variables in the model are defined to be the effective average values of the corresponding process variables. Since the dynamics of the economizer are negligible for the overall boiler dynamic performance, the economizer will not be included in the model.

A. Gas Path Equations

Time constants for gas dynamics are so short in comparison to the steam-water side (6) that the dynamics of the flue gas can be ignored. The heat produced in the combustion is determined by

$$q = K_f W_f + C_f W_f T_{fl} + C_a W_a T_a - C_{ash} W_{ash} T_{ash} \cdot$$

heat produced   heat carried   heat carried   heat lost

by fuel burning   by fuel   by air   with ash

$$(2-A1)$$

The mass rate of flue gas production is

$$W_g = W_f + W_a - W_{ash} \cdot \qquad (2-A2)$$

The heat produced in the furnace is absorbed by different boiler sections or lost to the air. An equation which describes the balance of energy of hot gas is

$$q = q_{gw} + q_p + q_s + q_f + q_{ge} + q_{gr} \qquad (2-A3)$$

where

q_{gw} = rate of heat transfer from gas to waterwall

q_p = rate of heat transfer to primary superheater

q_s = rate of heat transfer to secondary superheater

q_f = rate of heat transfer to finishing superheater

q_{gr} = flow of heat into reheater section

q_{ge} = flow of heat into economizer section

K_f = caloric value of fuel

C_f = specific heat of fuel

C_a = specific heat of air

C_{ash} = specific heat of ash

W_f = fuel flow rate

W_a = air flow rate

W_{ash} = rate of ash formation

T_{fl} = temperature of the fuel flowing into furnace

T_a = temperature of the air flowing into furnace

T_{ash} = ash temperature

and

$$q_{gr} = K_g W_g T_{gr} \qquad\qquad (2\text{-}A4)$$

$$q_{ge} = (1 - K_g)W_g T_{ge} \qquad\qquad (2\text{-}A5)$$

where

K_g = the fraction of gas which flows into reheater section

T_{gr} = temperature of the gas flowing into reheater section

T_{ge} = temperature of the gas flowing into economizer section

W_g = flow rate of the hot gas.

Defining

$$q_{sh} = q_p + q_s + q_f$$

$$q_{re} = q_{gr} + q_{ge}$$

and let q_n be the rate of heat flowing out of the furnace. The heat input and output relation in the gas path is then

$$q_n = q - q_{gw}$$

$$q_{re} = q_n - q_{sh}.$$

The incremental equations for these two equations are

$$\Delta q_n = \Delta q - \Delta q_{gw} \quad \text{and} \tag{2-A6}$$

$$\Delta q_{re} = \Delta q_n - \Delta q_{sh} \quad . \tag{2-A7}$$

The incremental equations for Eqs. (2-A4) and (2-A5) are

$$\frac{\Delta q_{gr}}{q_{gro}} = \frac{\Delta W_g}{W_{go}} + \frac{\Delta T_{gr}}{T_{gro}} \quad \text{and} \tag{2-A8}$$

$$\frac{\Delta q_{ge}}{q_{geo}} = \frac{\Delta W_g}{W_{go}} + \frac{\Delta T_{ge}}{T_{geo}} \quad . \tag{2-A9}$$

For complete fuel burning in combustion, the average fraction of ash produced from fuel is determined by the chemical property of the fuel. The average ratio of ash production rate and flow rate is a constant. Let

$$R_{hf} = \frac{W_{ash}}{W_f} \quad .$$

Equation (2-A2) becomes

$$W_g = (1 - R_{hf})W_f + W_a \tag{2-A10}$$

The incremental equation is simply

$$\Delta W_g = (1 - R_{hf})\Delta W_f + \Delta W_a \tag{2-A11}$$

Also, Eq. (2-A1) becomes

$$q = (K_f + C_f T_{fl} - C_{ash} T_{ash} R_{hf})W_f + C_a T_a W_a \tag{2-A12}$$

The incremental equation for this equation is

$$\Delta q = K \Delta W_f + C_a T_a \Delta W_a \qquad (2\text{-A}13)$$

where

$$K = K_f + C_f T_{f1} - C_{ash} T_{ash} R_{hf} \quad .$$

The temperature of flue gas in the furnace is estimated by

$$T_f = \frac{q}{C_g W_g} \quad .$$

The incremental form of this equation is

$$\Delta T_f = \frac{T_{fo}}{q_o} \Delta q - \frac{T_{fo}}{W_{go}} \Delta W_g \quad . \qquad (2\text{-A}14)$$

B. Transfer of Heat From Flue Gas to the Waterwall

Transfer of heat to the waterwall tubes involves a radiation process and a convection process. Since the convection heat transfer contributes only about six to seven percent of the total heat transfer to the waterwall tubes (28), the average heat transfer may be described by a radiation process only.

The radiant heat is absorbed by the tube wall in accordance with the Stefan-Boltzman Law:

$$q_{gw} = \sigma e A_f (T_f^4 - T_1^4) \qquad (2\text{-B}1)$$

where

T_f = flame temperature

T_1 = effective average tube wall temperature

e = emissivity factor, depending on the tube material and surface condition

σ = Stefan-Boltzman constant, 1.73×10^{-9} Btu/ft^{2}Hr$^\circ$R^4
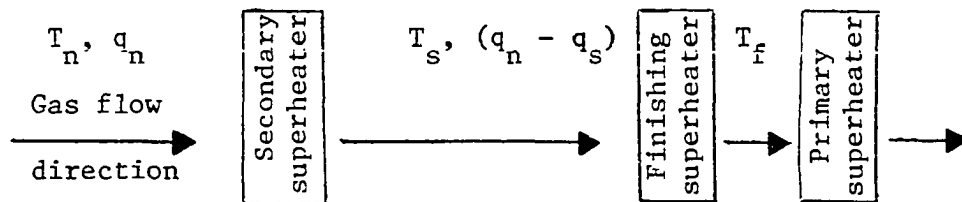
A_f = effective flame envelope area

q_{gw} = rate of heat transfer to waterwall

The incremental equation for (2-B1) is

$$\frac{\Delta q_{gw}}{q_{gwo}} = \frac{4T_{fo}^3}{T_{fo}^4 - T_{lo}^4} \Delta T_f - \frac{4T_{lo}^3}{T_{fo}^4 - T_{lo}^4} \Delta T_1 \qquad (2-B2)$$

C. Transfer of Heat From Flue Gas to Superheater

The flow of gas through the superheaters is in the following sequence

where the secondary superheater is also called a partial division wall

superheater. The finishing superheater is also called a pendant superheater,

and the primary superheater is also called a convection superheater.

The temperature of the flue gas leaving the furnace is given by

$$T_n = \frac{q - q_{gw}}{C_g \bar{W}_g} \qquad (2-C1)$$

The transfer of heat to the secondary superheater is by both radiation and

convection.

$$q_s = q_{sr} + q_{sc} \qquad (2-C2)$$

The rate of radiation heat transfer is given by

$$q_{sr} = \sigma e_1 A_1 (T_n^4 - T_{ms}^4) \qquad (2-C3)$$

The rate of convection heat transfer is given by

$$q_{sc} = K_1 W_g^n (T_n - T_{ms}) \tag{2-C4}$$

where

T_{ms} = effective average temperature of secondary superheater tube

wall surface

W_g = flue gas flow rate

C_g = specific heat of flow gas

The temperature of the flue gas leaving the secondary superheater is given

by

$$T_s = T_n - \frac{q_s}{C_g W_g} \tag{2-C5}$$

The transfer of heat to the finishing superheater is also by radiation and

convection. The corresponding equations are

$$q_f = q_{fr} + q_{fc} \tag{2-C6}$$

$$q_{fr} = \sigma e_2 A_2 (T_s^4 - T_{mf}^4) \tag{2-C7}$$

$$q_{fc} = K_2 W_g^n (T_s - T_{mf}) \tag{2-C8}$$

The transfer of heat to the primary superheater is mainly by convection.

The temperature of the flue gas leaving the finishing superheater is given

by

$$T_p = T_s - \frac{q_f}{C_g K_g W_g} \tag{2-C9}$$

where

K_g = the fraction of flue gas flowing into primary superheater

section

The heat transferred to the tube wall is

$$q_p = K_3(K_g W_g)^n (T_p - T_{mp}) \tag{2-C10}$$

The temperature of the gas flowing into the economizer section is given by

$$T_{ge} = T_p - \frac{q_p}{C_g K_g W_g}$$

The temperature of the flue gas flowing into the reheater section is

$$T_{gr} = T_p$$

and the fraction of gas which flows into the reheater section is

$$W_{re} = (1 - K_g) W_g \quad .$$

The total heat flowing into the reheater section and the economizer section is

$$q_{re} = q_{gr} + q_{ge} = K_g W_g T_{gp} + (1 - K_g) W_g T_{ge}$$

where the variables have already been defined in Section A for Eqs. (2-A4) and (2-A5).

The incremental equations for the above equations are

$$\frac{\Delta T_n}{T_{no}} = \frac{1}{q_o - q_{gwo}} \Delta q - \frac{1}{q_o - q_{gwo}} \Delta q_{gw} - \frac{\Delta W_g}{W_{go}} \tag{2-C1a}$$

$$\Delta q_s = \Delta q_{sr} + \Delta q_{sc} \tag{2-C2a}$$

$$\frac{\Delta q_{sr}}{q_{sro}} = \frac{4 T_{no}^3}{T_{no}^4 - T_{mso}} \Delta T_n - \frac{4 T_{mso}^3}{T_{no}^4 - T_{mso}^4} \Delta T_{ms} \tag{2-C3a}$$

$$\frac{\Delta q_{sc}}{q_{sco}} = \frac{1}{T_{no} - T_{mso}} \Delta T_n - \frac{1}{T_{no} - T_{mso}} \Delta T_{ms} + \frac{n}{W_{go}} \Delta W_g \tag{2-C4a}$$

$$\Delta T_s = \Delta T_n - \frac{T_{no} - T_{so}}{q_{so}} \Delta q_s + \frac{T_{no} - T_{so}}{W_{go}} \Delta W_g \qquad (2\text{-}C5a)$$

$$\Delta q_f = \Delta q_{fr} + \Delta q_{fc} \qquad (2\text{-}C6a)$$

$$\frac{\Delta q_{fr}}{q_{fro}} = \frac{4T_{so}^3}{T_{so}^4 - T_{mfo}^4} \Delta T_s \quad \frac{4T_{mfo}^3}{T_{so}^4 - T_{mfo}^4} \Delta T_{mf} \qquad (2\text{-}C7a)$$

$$\frac{\Delta q_{fc}}{q_{fco}} = \frac{1}{T_{so} - T_{mfo}} \Delta T_s - \frac{1}{T_{so} - T_{mfo}} \Delta T_{mf} + \frac{n}{W_{go}} \Delta W_g \qquad (2\text{-}C8a)$$

$$\Delta T_q = \Delta T_s - \frac{T_{so} - T_{po}}{q_{fo}} \Delta q_f + \frac{T_{so} - T_{po}}{W_{go}} \Delta W_g \qquad (2\text{-}C9a)$$

$$\frac{\Delta q_p}{q_{po}} = \frac{1}{T_{po} - T_{mpo}} \Delta T_p - \frac{1}{T_{po} - T_{mpo}} \Delta T_{mp} + \frac{n}{W_{go}} \Delta W_g \qquad (2\text{-}C10a)$$

Combining Eqs. (2-C2a), (2-C3a), and (2-C4a),

$$\Delta q_s = \left(\frac{4q_{sro} T_{no}^3}{T_{no}^4 - T_{mso}^4} + \frac{q_{sco}}{T_{no} - T_{mso}} \right) \Delta T_n$$

$$- \left(\frac{4q_{sro} T_{mso}^3}{T_{no}^4 - T_{mso}^4} + \frac{q_{sco}}{T_{no} - T_{mso}} \right) \Delta T_{ms} + \frac{nq_{sco}}{W_{go}} \Delta W_g \qquad (2\text{-}C11)$$

Combining Eqs. (2-C6a), (2-C7a), and (2-C8a),

$$\Delta q_f = \left(\frac{4q_{fro} T_{so}^3}{T_{so}^4 - T_{mfo}^4} + \frac{q_{fco}}{T_{so} - T_{mfo}} \right) \Delta T_s$$

$$- \left(\frac{4q_{fro} T_{mfo}^3}{T_{so}^4 - T_{mfo}^4} + \frac{q_{fco}}{T_{so} - T_{mfo}} \right) \Delta T_{mf} + \frac{nq_{fco}}{W_{go}} \Delta W_g \qquad (2\text{-}C12)$$

Substituting Eq. (2-C1a) into Eq. (2-C5a),

$$\Delta T_s = \frac{T_{no}}{q_o - q_{gwo}} \Delta q - \frac{T_{no}}{q_o - q_{gwo}} \Delta q_{gw} - \frac{T_{no} - T_{so}}{q_{so}} \Delta q_s - \frac{T_{so}}{W_{go}} \Delta W_g$$

(2-C13)

Substituting Eq. (2-C13) into Eq. (2-C9a),

$$\Delta T_p = \frac{T_{no}}{q_o - q_{wo}} \Delta q - \frac{T_{no}}{q_o - q_{gwo}} \Delta q_{gw} - \frac{T_{no} - T_{so}}{q_{so}} \Delta q_s$$

$$- \frac{T_{so} - T_{po}}{q_{fo}} \Delta q_f - \frac{T_{po}}{W_{go}} \Delta W_g$$

(2-C14)

Substituting Eq. (2-C1a) into Eq. (2-C11),

$$\Delta q_s = K_1(\Delta q - \Delta q_{gw}) - K_2 \Delta T_{ms} - K_3 \Delta W_g$$

(2-C15)

where

$$K_1 = \frac{T_{no}}{q_o - q_{gwo}} \left(\frac{4 q_{sro} T_{no}^3}{T_{no}^4 - T_{mso}^4} + \frac{q_{sco}}{T_{no} - T_{mso}} \right)$$

$$K_2 = \frac{4 q_{sro} T_{mso}}{T_{no}^4 - T_{mso}^4} + \frac{q_{sco}}{T_{no} - T_{mso}}$$

$$K_3 = \frac{T_{no}}{W_{go}} \left(\frac{4 q_{sro} T_{no}^3}{T_{no}^4 - T_{mso}^4} + \frac{q_{sco}}{T_{no} - T_{mso}} \right) - \frac{n q_{sco}}{W_{go}}$$

Substituting Eq. (2-C5) into Eq. (2-C13),

$$\Delta T_s = \left(\frac{T_{no}}{q_o - q_{gwo}} - K_1 \frac{T_{no} - T_{so}}{q_{so}} \right) \left(\Delta q - \Delta q_{gw} \right)$$

$$+ K_2 \frac{T_{no} - T_{so}}{q_{so}} \Delta T_{ms} - \left(\frac{T_{so}}{W_{go}} + K_3 \frac{T_{no} - T_{so}}{q_{so}} \right) \Delta W_g$$

(2-C16)

Substituting Eq. (2-C16) into Eq. (2-C12),

$$\Delta q_f = K_4(\Delta q - \Delta q_{gw}) + K_5 \Delta T_{ms} - K_6 \Delta T_{mf} - K_7 \Delta W_g$$

(2-C17)

where

$$K_4 = \left(\frac{4q_{fro}T_{so}^3}{T_{so}^4 - T_{mfo}^4} + \frac{q_{fco}}{T_{so} - T_{mfo}} \right) \left(\frac{T_{no}}{q_o - q_{gwo}} - K_1 \frac{T_{no} - T_{so}}{q_{so}} \right)$$

$$K_5 = K_2 \left(\frac{4q_{fro}T_{so}^3}{T_{so}^4 - T_{mfo}^4} + \frac{q_{fco}}{T_{so} - T_{mfo}} \right) \left(\frac{T_{no} - T_{so}}{q_{so}} \right)$$

$$K_6 = \frac{4q_{fro}T_{mfo}^3}{T_{so}^4 - T_{mfo}^4} + \frac{q_{fco}}{T_{so} - T_{mfo}}$$

$$K_7 = \left(\frac{4q_{fro}T_{so}^3}{T_{so}^4 - T_{mfo}^4} + \frac{q_{fco}}{T_{so} - T_{mfo}} \right) \left(\frac{T_{so}}{W_{go}} + K_3 \frac{T_{no} - T_{so}}{q_{so}} \right) - \frac{nq_{fco}}{W_{go}}$$

Substituting Eqs. (2-C15) and (2-C17) into Eq. (2-C14),

$$\Delta T_p = \left(\frac{T_{no}}{q_o - q_{gwo}} - K_1 \frac{T_{no} - T_{so}}{q_{so}} - K_4 \frac{T_{so} - T_{po}}{q_{fo}} \right) (\Delta q - \Delta q_{gw})$$

$$+ \left(K_2 \frac{T_{no} - T_{so}}{q_{so}} - K_5 \frac{T_{so} - T_{po}}{q_{fo}} \right) \Delta T_{ms} + K_6 \left(\frac{T_{so} - T_{po}}{q_{fo}} \right) \Delta T_{mf}$$

$$- \left(\frac{T_{po}}{W_{go}} - K_3 \frac{T_{no} - T_{so}}{q_{so}} - K_7 \frac{T_{so} - T_{po}}{q_{fo}} \right) \Delta W_g \qquad \text{(2-C18)}$$

Substituting Eq. (2-C18) into Eq. (2-C10a),

$$\Delta q_p = K_8 (\Delta q - \Delta q_{gw}) + K_9 \Delta T_{ms} + K_{10} \Delta T_{mf} - K_{11} \Delta T_{mp} - K_{12} \Delta W_g \qquad \text{(2-C19)}$$

where

$$K_8 = \frac{q_{po}}{T_{fo} - T_{mpo}} \left(\frac{T_{no}}{q_o - q_{gwo}} - K_1 \frac{T_{no} - T_{so}}{q_{so}} - K_4 \frac{T_{so} - T_{po}}{q_{fo}} \right)$$

$$K_9 = \frac{q_{po}}{T_{po} - T_{mpo}} \left(K_2 \frac{T_{no} - T_{so}}{q_{so}} - K_5 \frac{T_{so} - T_{po}}{q_{fo}} \right)$$

$$K_{10} = \frac{K_6 q_{po}}{T_p - T_{mpo}} \left(\frac{T_{so} - T_{po}}{q_{fo}} \right)$$

$$K_{11} = \frac{q_{po}}{T_{po} - T_{mpo}}$$

$$K_{12} = \frac{q_{po}}{T_{po} - T_{mpo}} \left(\frac{T_{po}}{W_{go}} - K_3 \frac{T_{no} - T_{so}}{q_{so}} - K_7 \frac{T_{so} - T_{po}}{q_{fo}} \right)$$

The total heat absorption in the superheater sections is

$$q_{sh} = q_s + q_f + q_p$$

Or, in incremental form

$$\Delta q_{sh} = \Delta q_s + \Delta q_f + \Delta q_p \qquad (2\text{-}C20)$$

Substituting Eqs. (2-C15), (2-C7), and (2-C19) into Eq. (2-C20)

$$\Delta q_{sh} = K_{14}(\Delta q - \Delta q_{gw}) - K_{15}\Delta T_{ms} - K_{16}\Delta T_{mf} - K_{11}\Delta T_{mp} - K_{17}\Delta W_g$$

$$(2\text{-}C21)$$

where

$$K_{14} = K_1 + K_4 + K_8$$

$$K_{15} = K_2 - K_5 - K_9$$

$$K_{16} = K_6 - K_{10}$$

$$K_{17} = K_3 + K_7 + K_{12}$$

D. Transfer of Heat Through Metal Tube Wall

Assume that the temperature on both the inner and outer surfaces of a metal tube are uniform. Then the transfer of heat through the tube can be approximated by the radial heat conduction. Shang (27) used thin layer approximation to represent the physical heat transfer process as the radial heat conduction, as shown in Fig. 4.
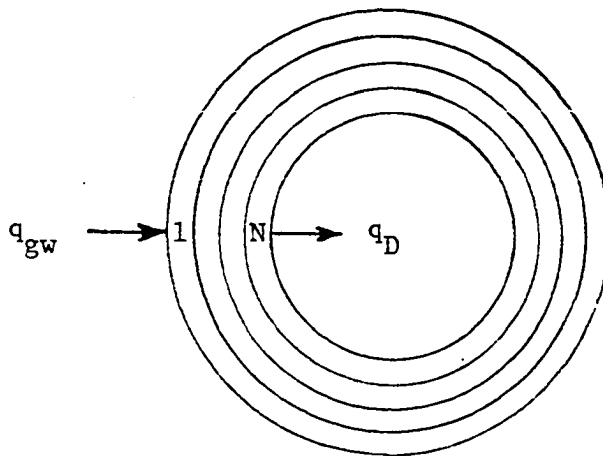
Fig. 4. A tube wall divided into thin layers.

The thin layer approximation can be used to represent a heat transfer delay through the tube wall. Suppose that the metal tube wall is composed of N thin layers so that transfer of heat in each thin layer is in the radial direction. Let

q_i = rate of heat flow out from ith layer and into $(i + 1)$th layer.

T_i = average metal temperature of each layer.

Then the variation of temperature in each layer is given by the equations

$$q_{gx} - q_1 = \rho_1 c_1 V_1 \frac{dT_1}{dt}$$

$$q_1 - q_2 = \rho_2 c_2 V_2 \frac{dT_2}{dt}$$

$$q_2 - q_3 = \rho_3 c_3 V_3 \frac{dT_3}{dt}$$

.
.
.

$$q_{N-1} - q_{Dx} = \rho_N c_N V_N \frac{dT_N}{dt} \qquad (2\text{-}D1)$$

and the transfer of heat between the layers is determined by

$$q_1 = A_1 h_1 (T_1 - T_2)$$

$$q_2 = A_2 h_2 (T_2 - T_3)$$

$$\vdots$$

$$q_{N-1} = A_{N-1} h_{N-1} (T_{N-1} - T_N) \qquad (2\text{-}D2)$$

where

A_1 = equivalent heat transfer area of the ith layer

$$= \frac{2\Pi L}{\ln \left(\frac{r_{i+1}}{r_i} \right)}$$

r_i = inner radius of the ith layer

L = effective tube length

ρ_i = metal density of the ith layer

c_i = specific heat of the ith layer

V_i = volume of the ith layer

h_i = heat transfer coefficient between the ith layer and the (i + 1)th layer

q_{gx} = rate of heat transfer between gas and the outer surface of tube of section x

q_{Dx} = rate of heat transfer between inner surface of the tube wall of the section x and the fluid inside the tube.

Combining the above two sets of equations, the state-space equations describing the temperature variations in the tube layers and obtained

$$\frac{dT_1}{dt} = -\frac{A_1 h_1}{\rho_1 c_1 V_1} T_1 + \frac{A_1 h_1}{\rho_1 c_1 V_1} T_2 + \frac{1}{\rho_1 c_1 V_1} q_{gx}$$

$$\frac{dT_2}{dt} = \frac{A_1 h_1}{\rho_2 c_2 V_2} T_1 - \frac{(A_1 h_1 + A_2 h_2)}{\rho_2 c_2 V_2} T_2 + \frac{A_2 h_2}{\rho_2 c_2 V_2} T_3$$

$$\frac{dT_3}{dt} = \frac{A_2 h_2}{\rho_3 c_3 V_3} T_2 - \frac{(A_2 h_2 + A_3 h_3)}{\rho_3 c_3 V_3} T_3 + \frac{A_3 h_3}{\rho_3 c_3 V_3} T_4$$

$$\vdots$$

$$\frac{dT_{N-1}}{dt} = \frac{A_{N-2} h_{N-2}}{\rho_{N-1} c_{N-1} V_{N-1}} T_{N-2} - \frac{(A_{N-2} h_{N-2} + A_{N-1} h_{N-1})}{\rho_{N-1} c_{N-1} V_{N-1}} T_{N-1} \qquad (2\text{-}D3)$$

$$+ \frac{A_{N-1} h_{N-1}}{\rho_{N-1} c_{N-1} V_{N-1}} T_N$$

$$\frac{dT_N}{dt} = \frac{A_{N-1} h_{N-1}}{\rho_N c_N V_N} T_{N-1} - \frac{A_{N-1} h_{N-1}}{\rho_N c_N V_N} T_N - \frac{1}{\rho_N c_N V_N} q_{Dx}$$

The incremental equations for (2-D3) are

$$\frac{d\Delta T_1}{dt} = \frac{A_1 h_1}{\rho_1 c_1 V_1} \Delta T_1 + \frac{A_1 h_1}{\rho_1 c_1 V_1} \Delta T_2 + \frac{1}{\rho_1 c_1 V_1} \Delta q_{gx}$$

$$\frac{d\Delta T_2}{dt} = \frac{A_1 h_1}{\rho_2 c_2 V_2} \Delta T_1 - \frac{(A_1 h_1 + A_2 h_2)}{\rho_2 c_2 V_2} \Delta T_2 + \frac{A_2 h_2}{\rho_2 c_2 V_2} \Delta T_3$$

$$\vdots$$

$$\frac{d\Delta T_{N-1}}{dt} = \frac{A_{N-2} h_{N-2}}{\rho_{N-1} c_{N-1} V_{N-1}} \Delta T_{N-2} - \frac{(A_{N-2} h_{N-2} + A_{N-1} h_{N-1})}{\rho_{N-1} c_{N-1} V_{N-1}} \Delta T_{N-1}$$

$$+ \frac{A_{N-1} h_{N-1}}{\rho_{N-1} c_{N-1} V_{N-1}} \Delta T_N \qquad (2\text{-}D4)$$

$$\frac{d\Delta T_N}{dt} = \frac{A_{N-1}h_{N-1}}{\rho_N c_N V_N} \Delta T_{N-1} - \frac{A_{N-1}h_{N-1}}{\rho_N c_N V_N} \Delta T_N - \frac{1}{\rho_N c_N V_N} \Delta q_{Dx}$$

E. Heat Transfer Between Inner Most Layers of Metal

Tube and Working Fluid Inside the Tube

For fluid flowing inside the tubes, the convective heat transfer rate may be described by

$$q_D = h A_n (T_N - T_D) \tag{2-E1}$$

with the heat transfer coefficient correlated by

$$\bar{N}u = \alpha(\bar{R}e^m)(Pr)^n \tag{2-E2}$$

where

Nu	= Nusselt number = hD/k
Re	= Reynolds number = UD/μ
Pr	= Prandtl number = $c_p \mu/k$
α, m, n	= experimental constants
D	= tube inner diameter
k	= thermal conductivity of the fluid
U	= W_D/A = rate of mass flow per unit cross-sectional tube area
μ	= absolute viscosity
c_p	= specific heat of the fluid at constant pressure
A_n	= heat transfer surface area
T_N	= temperature of the inner most tube layer
T_D	= average bulk fluid temperature
W_D	= average fluid mass flow rate

Substituting Nu and Re into Eq. (2-E2) and solving for h:

$$h = \left(\frac{k}{D}\right)\alpha\left(\frac{W_D}{A}\ \frac{D}{\mu}\right)^m (Pr)^n$$

Let

$$A_n A^{-m} D^{m-1} = const$$

$$\alpha k \mu^{-m} = \phi$$

then

$$h = const\ \phi\ (W_D)^m (Pr)^n (A_n)^{-1}$$

and

$$q_D = const\ \phi\ (W_D)^{\overline{m}} (Pr)^{\overline{n}} (T_N - T_D) \qquad (2\text{-}E3)$$

The development above follows that given by Shang (27). The incremental equation for Eq. (2-E3) is

$$\frac{\Delta q_D}{q_{Do}} = \frac{\Delta\phi}{\phi_o} + m\ \frac{\Delta W_D}{W_{Do}} + n\ \frac{\Delta Pr}{(Pr)_o} + \frac{\Delta T_N - \Delta T_D}{T_{No} - T_{Do}} \qquad (2\text{-}E4)$$

where ϕ and Pr are functions of pressure and temperature and may be represented in terms of these variables.

$$\frac{\Delta q_D}{q_{Do}} = \frac{1}{\phi_o}\left(\frac{\partial\phi}{\partial D_D}\ \Delta D_D + \frac{\partial\phi}{\partial T_D}\ \Delta T_D\right) + m\ \frac{\Delta W_D}{W_{Do}}$$

$$+ \frac{n}{(Pr)_o}\left(\frac{\partial Pr}{\partial D_D}\ \Delta D_D + \frac{\partial Pr}{\partial T_D}\ \Delta T_D\right) + \frac{\Delta T_N - \Delta T_D}{T_{No} - T_{Do}}$$

$$= \frac{m}{W_{Do}}\ \Delta W_D + \frac{1}{T_{No} - T_{Do}}\ \Delta T_N + \left(\frac{1}{\phi_o}\ \frac{\partial\phi}{\partial D_D} + \frac{n}{(Pr)_o}\ \frac{\partial Pr}{\partial D_D}\right)\Delta D_D$$

$$+ \left(\frac{1}{\phi_o}\ \frac{\partial\phi}{\partial T_D} + \frac{n}{(Pr)_o}\ \frac{\partial Pr}{\partial T_D} - \frac{1}{T_{No} - T_{Do}}\right)\Delta T_D \qquad (2\text{-}E4a)$$

Equation (2-E3) is applicable for the superheater and the waterwall

sections. For the superheater sections, Eq. (2-E4a) becomes

$$\Delta q_{Dsh} = K_{13}\Delta W_v + K_{14}\Delta T_m + K_{15}\Delta D_d - K_{16}\Delta T_{sh} \qquad (2-E5)$$

where

Δq_{Dsh} = heat absorbed by the steam in the respective superheater

ΔW_v = rate of steam flow in the respective superheater

ΔT_{sh} = temperature of steam in the respective superheater

$$K_{13} = \frac{mq_{Do}}{W_{Do}} = \frac{mq_{Dsho}}{W_{vo}} \qquad \text{for respective superheater}$$

$$K_{14} = \frac{q_{Do}}{T_{No} - T_{Do}} = \frac{q_{Dsho}}{T_{mo} - T_{sho}} \qquad \text{for respective superheater}$$

$$K_{15} = \left(\frac{1}{\phi_o}\frac{\partial \phi}{\partial D_D} + \frac{n}{(Pr)_o}\frac{\partial Pr}{\partial D_D} \right) q_{Do}$$

$$= \left(\frac{1}{\phi_o}\frac{\partial \phi}{\partial D_D} + \frac{n}{(Pr)_o}\frac{\partial Pr}{\partial D_D} \right) q_{sho} \qquad \text{for respective superheater}$$

$$K_{16} = \left(\frac{1}{T_{No} - T_{Do}} - \frac{n}{(Pr)_o}\frac{\partial Pr}{\partial T_D} - \frac{1}{\phi_c}\frac{\partial \psi}{\partial D_D} \right) q_{Do}$$

$$= \left(\frac{1}{T_{mo} - T_{sho}} - \frac{n}{(Pr)_o}\frac{\partial Pr}{\partial T_{sh}} - \frac{1}{\phi_o}\frac{\partial \phi}{\partial D_D} \right) q_{sho} \qquad \text{for respective superheater}$$

In the waterwall section the fluid temperature and pressure are

related by Clapeyron equation

$$\frac{\Delta D_D}{\Delta T_D} = \frac{h_{fgo}}{T_{Do}v_{fgo}} \quad .$$

The rate of heat transfer to the steam-water mixture in the tube may be

written as

$$\Delta q_{DW} = K_{18}\Delta W_D + K_{19}\Delta T_N - K_{20}\Delta D_D \qquad (2\text{-}E6)$$

where

$$K_{18} = \frac{m q_{DWo}}{W_{Do}}$$

$$K_{19} = \frac{q_{DWo}}{T_{No} - T_{Do}}$$

$$K_{20} = \frac{T_{Do} v_{fgo}}{h_{fgo}} \left(\frac{1}{T_{No} - T_{Do}} - \frac{n}{(Pr)_o} \frac{\partial Pr}{\partial T_D} - \frac{1}{\phi_o} \frac{\partial \phi}{T_D} \right)$$

$$- \left(\frac{1}{\phi_o} \frac{\partial \phi}{\partial D_D} + \frac{n}{(Pr)_o} \frac{\partial Pr}{\partial D_D} \right)$$

F. Variation of Drum Pressure

Assume that (3)

a) water in the drum, downcomer, and riser are saturated water at drum pressure, and

b) steam in the drum and in the riser tube is saturated steam at drum pressure.

Let

W_{fw} = feedwater flow rate,

W_v = rate of steam flow out from drum,

V_{wd} = volume of water in drum, downcomer, and riser,

V_{gd} = volume of steam in drum, downcomer, and riser, and

D_D = drum pressure ,

where

$$V_{wd} + V_{gd} = V = \text{total internal volume of drum, downcomer, and}$$
$$\text{riser.}$$

Then the mass balance equation is

$$W_{fw} - W_v = \frac{d}{dt}(\rho_f V_{wd} + \rho_g V_{gd}) = \frac{d}{dt}[\rho_f V_{wd} + \rho_g(v - V_{wd})]$$

where

ρ_f = density of the water in drum, and

ρ_g = density of the steam in drum.

The incremental equation for Eq. (2-F1) is

$$\Delta W_{fw} - \Delta W_v = \frac{d}{dt}\left[(\rho_{fo} - \rho_{go})\Delta V_{wd} + V_{wdo}\Delta\rho_f + V_{gdo}\Delta\rho_g\right]$$

$$= (\rho_{fo} - \rho_{go})\frac{d\Delta V_{wd}}{dt} + V_{wdo}\frac{d\Delta\rho_f}{dt} + V_{gdo}\frac{d\Delta\rho_g}{dt} \qquad (2\text{-F2})$$

Since ρ_f and ρ_g are functions of drum pressure, Eq. (2-F2) may be written as

$$\Delta W_{fw} - \Delta W_v = (\rho_{fo} - \rho_{go})\frac{d\Delta V_{wd}}{dt} + \left[V_{wdo}\frac{\partial\rho_f}{\partial D_D} + V_{gdo}\frac{\partial\rho_g}{\partial D_D}\right]\frac{d\Delta D_D}{dt}$$

$$(2\text{-F3})$$

The energy balance equation is

$$h_e W_{fw} - h_g W_v + q_{Dw} = \frac{d}{dt}(h_f \rho_f V_{wd} + h_g \rho_g V_{gd}) \qquad (2\text{-F4})$$

where

h_e = enthalphy of feedwater

h_f = enthalphy of water in drum

h_g = enthalphy of steam in drum

The linear incremental equation for Eq. (2-F4) is

$$h_e \Delta W_{fw} = h_{go}\Delta W_v - W_{vo}\Delta h_g + \Delta_q Dw$$

$$= \frac{d}{dt}[h_{fo}\rho_{fo}\Delta V_{wd} + h_{fo}V_{wdo}\Delta\rho_f + \rho_{fo}V_{wdo}\Delta h_f$$

$$+ h_{go}\rho_{go}\Delta V_{gd} + h_{go}V_{gdo}\Delta\rho_g + V_{gdo}\rho_{go}\Delta h_g] \qquad (2\text{-F5})$$

Since h_f, h_g, ρ_f, and ρ_g are functions of pressure in the drum, Eq. (2-F5) can be written as

$$h_e \Delta W_{fw} - h_{go} \Delta W_v + \Delta q_{Dw} - W_{vo} \frac{\partial h_g}{\partial D_d} \Delta D_D$$

$$= (h_{fo}\rho_{fo} - h_{go}\rho_{go}) \frac{d\Delta V_{Wd}}{dt} + \left[h_{fo} V_{wdo} \frac{\partial \rho_f}{\partial D_D} + \rho_{fo} V_{wdo} \frac{\partial h_f}{\partial D_D} \right.$$

$$\left. + h_{go} V_{gdo} \frac{\partial \rho_g}{\partial D_D} + V_{gdo}\rho_{go} \frac{\partial h_g}{\partial D_D} \right] \frac{dD_D}{dt} \qquad (2\text{-}F6)$$

where

$$\frac{dV_{gd}}{dt} = \frac{d}{dt} (V - V_{wd}) = - \frac{dV_{wd}}{dt} \qquad .$$

Let

$$a_1 = \rho_{fo} - \rho_{go}$$

$$a_2 = V_{wdo} \frac{\partial \Delta \rho_f}{\partial D_D} + V_{gdo} \frac{\partial \Delta \rho_g}{\partial D_D}$$

$$a_3 = h_{fo}\rho_{fo} - h_{go}\rho_{go}$$

$$a_4 = h_{fo} V_{wdo} \frac{\partial \rho_f}{\partial D_D} + \rho_{fo} V_{wdo} \frac{\partial h_f}{\partial D_D} + h_{go} V_{go} \frac{\partial \rho_g}{\partial D_D} + V_{gdo}\rho_{go} \frac{\partial h_g}{\partial D_D}$$

$$a_5 = W_{vo} \frac{\partial h_g}{\partial D_D}$$

Then Eq. (2-F3) becomes

$$a_1 \frac{d\Delta V_{wd}}{dt} + a_2 \frac{d\Delta D_D}{dt} = \Delta W_{fw} - \Delta W_v \qquad (2\text{-}F7)$$

Equation (2-F6) becomes

$$a_3 \frac{d\Delta V_{wd}}{dt} + a_4 \frac{d\Delta D_D}{dt} + a_5 \Delta D_D = h_e \Delta W_{fw} - h_{go} \Delta W_v + \Delta q_{Dw} \,. \qquad (2\text{-}F8)$$

Combining Eqs. (2-F7) and (2-F8) and eliminating ΔV_{wd}:

$$(a_1 a_4 - a_2 a_3) \frac{d\Delta D_D}{dt} + a_1 a_5 \Delta D_D = (a_1 h_e - a_3)\Delta W_{fw} - (a_1 h_{go} - a_3)\Delta W_v$$

$$+ a_1 \Delta q_{Dw}$$

or

$$\frac{d\Delta D_D}{dt} + \frac{a_1 a_5}{a_1 a_4 - a_2 a_3} \Delta D_D$$

$$= \frac{a_1 h_e - a_3}{a_1 a_4 - a_2 a_3} \Delta W_{fw} - \frac{a_1 h_{go} - a_3}{a_1 a_4 - a_2 a_3} \Delta W_v + \frac{a_1 \Delta q_{Dw}}{a_1 a_4 - a_2 a_3} \qquad (2\text{-}F9)$$

G. Superheater Equations

Assume that the pressure drop across the superheater is negligible.

Then the steam pressure in the drum may be represented by the drum pressure.

The superheater equations may be written as (5,11,15)

$$W_a - W_b = V \frac{d\rho_b}{dt} \qquad (2\text{-}G1)$$

$$q_{st} + W_a h_a - W_b h_b = V \frac{d}{dt} (\rho_b h_b) \qquad (2\text{-}G2)$$

where

W_a = rate of steam flow into the superheater section

W_b = rate of steam flow out from the superheater section

V = superheater volume

ρ_b = steam density at superheater outlet

h_b = enthalphy of steam at superheater outlet

h_a = enthalphy of steam at superheater inlet

q_{st} = rate of flow of heat from tube wall to steam

The incremental equation for Eq. (2-G1) is

$$\Delta W_a - \Delta W_b = V \frac{d\Delta\rho_b}{dt} \quad .$$

(2-G3)

The incremental equation for Eq. (2-G2) is

$$\Delta q_{st} + W_{ao}\Delta h_a + h_{ao}\Delta W_a - W_{bo}\Delta h_b - h_{bo}\Delta W_b$$

$$= V \left(\rho_{bo} \frac{d\Delta h_b}{dt} + h_{bo} \frac{d\Delta\rho_b}{dt} \right) .$$

(2-G4)

Combining Eqs. (2-G3) and (2-G4)

$$\Delta q_{st} + W_{ao}\Delta h_a - W_{bo}\Delta h_b - (h_{bo} - h_{ao})\Delta W_b$$

$$= V \left(\rho_{bo} \frac{d\Delta h_b}{dt} + (h_{bo} - h_{ao}) \frac{d\Delta\rho_b}{dt} \right)$$

(2-G5)

Since h_a, h_b, and ρ_b are functions of pressure and temperature, Eq. (2-G5) can be written as

$$\Delta q_{st} + \left(W_{ao} \frac{\partial h_a}{\partial D_D} - W_{bo} \frac{\partial h_b}{\partial D_D} \right) \Delta D_D + W_{ao} \frac{\partial h_a}{\partial T_a} \Delta T_a - W_{bo} \frac{\partial h_b}{\partial T_b} \Delta T_b$$

$$- (h_{bo} - h_{ao})\Delta W_b = V \left[\rho_{bo} \frac{\partial h_b}{\partial D_D} + (h_{bo} - h_{ao}) \frac{\partial \rho_b}{\partial D_D} \right] \frac{d\Delta D_D}{dt}$$

$$+ \left[V \rho_{bo} \frac{\partial h_b}{\partial T_b} + (h_{bo} - h_{ao}) \frac{\partial \rho_b}{\partial T_b} \right] \frac{d\Delta T_b}{dt}$$

(2-G6)

Let

$$b_1 = V \left[\rho_{bo} \frac{\partial h_b}{\partial T_b} + (h_{bo} - h_{ao}) \frac{\partial \rho_b}{\partial T_b} \right]$$

$$b_2 = W_{bo} \frac{\partial h_b}{\partial T_b}$$

$$b_3 = -V\left[\rho_{bo} \frac{\partial h_b}{\partial D_D} + (h_{bo} - h_{ao}) \frac{\partial \rho_b}{\partial D_D}\right]$$

$$b_4 = W_{bo} \frac{\partial h_b}{\partial D_D} - W_{ao} \frac{\partial h_a}{\partial D_D}$$

$$b_5 = W_{ao} \frac{\partial h_a}{\partial T_a}$$

$$b_6 = h_{bo} - h_{ao}$$

Equation (2-G6) can be expressed as

$$b_1 \frac{d\Delta T_b}{dt} + b_2 \Delta T_b = b_3 \frac{d\Delta D_D}{dt} - b_4 \Delta D_D + b_5 \Delta T_a - b_6 \Delta W_b + \Delta q_{st} \qquad (2\text{-}G7)$$

H. Flow of Superheated Steam Through Throttle Valves

The equation for estimating the rate of steam flow through the throttle valves has been given in many books on steam turbines (14).

$$W_T = C_T A_T \sqrt{\frac{D_T}{v_T}} \qquad (2\text{-}H1)$$

where

D_T = throttle pressure

v_T = specific volume of the steam before throttle

A_T = effective throttle area

W_T = steam flow rate

$$C_T = \left[\frac{2g\eta m}{m-1}\left(\gamma^{2/m} - \gamma^{(m+1)/m}\right)\right]^{1/2}$$

$$m = \frac{n}{n - \eta(n-1)}$$

η = valve efficiency

γ = pressure ratio

n = adiabatic index

For superheated steam, the ideal gas equation is observed

$$D_T v_T = RT_T$$

or

$$v_T = \frac{RT_T}{D_T} \qquad (2-H2)$$

Substituting Eq. (2-H2) into (2-H1),

$$W_T = \frac{C_T}{\sqrt{R}} A_T \frac{D_T}{\sqrt{T_T}} \triangleq C_T' A_T \frac{D_T}{\sqrt{T_T}} \qquad (2-H3)$$

The incremental equation for Eq. (2-H3) is

$$\frac{\Delta W_T}{W_{To}} = \frac{\Delta A_T}{A_{To}} + \frac{\Delta D_T}{D_{To}} - \frac{1}{2} \frac{\Delta T_T}{T_{To}} \qquad (2-H4)$$

Another equation which predicts the steam flow rate is Napier's experimental equation

$$W_T = CA_T D_T \qquad (2-H5)$$

The incremental equation for this equation is

$$\frac{\Delta W_T}{W_{To}} = \frac{\Delta A_T}{A_{To}} + \frac{\Delta D_T}{D_{To}} \qquad . \qquad (2-H6)$$

III. BOILER MODEL

The boiler model will be presented in the form of block diagrams in
the complex frequency domain. This kind of presentation is to show the
model in compact form, which provides better visualization of the relation-
ship among the boiler variables. The boiler models developed with the
black-box approach are usually presented in this form. The model developed
here may be classified as a gray-box model because the model equations in
the "box" are developed from physical principles.

The boiler variables which are important for control studies have been
included in the model. The model has five inputs and four outputs. The
inputs are

1) fuel flow rate,

2) air flow rate,

3) feedwater flow rate,

4) control valve area, and

5) flow rate of the circulation fluid.

The outputs are

1) steam flow rate,

2) drum pressure,

3) throttle steam temperature, and

4) the heat flow into reheater and economizer sections.

The heat flow into the reheater and the economizer sections is the
heat lost from the system considered for the model. This output may not be
controlled, but the physical data for this heat loss are required for param-
eter identification computation. The steam flow rate is the main variable

to be controlled since it determines the amount of steam energy flowing into the turbine. Drum pressure and throttle steam temperature are the most significant factors which influence the dynamic properties of a boiler. For efficient operation of a boiler, it is desirable to have these two variables controlled so that the variation of these variables are minimum during load variations.

The model has included the following boiler variables as state variables:

a) waterwall tube metal temperature,

b) primary superheater metal temperature,

c) secondary superheater metal temperature,

d) finishing superheater metal temperature,

e) outlet steam temperature of primary superheater, and

f) outlet steam temperature of secondary superheater.

For safe operation of a boiler, it is important to maintain these temperature values below the safety margins of the respective boiler components. The superheater outlet temperatures are usually controlled by superheater sprays. The limitation of metal temperatures may be included in the constraint functions for control studies.

In this chapter, a complete boiler model and a simplified boiler model in which the superheater sections are treated as one superheater are presented. The metal wall of the boiler is treated as one single layer. However, there is no difficulty in obtaining the transfer function with the tube wall divided into more layers; it only takes more time for mathematical manipulation. The transfer functions with the tube walls divided into

three layers are given in Appendix A and B. It is not clear what is the appropriate number of layers to be used to describe the delays of heat transfer through the tube walls. This can be determined with the results of the parameter identification computation. At the beginning, the wall may be treated as one single layer. If the heat transfer delay is not properly represented, the error between the model outputs and corresponding physical data will be large. Then the number of layers should be increased.

A. Model Equations

The boiler process equations have been developed in Chapter II. The equations which contribute directly to the composition of a boiler model are collected here.

a) Gas Path

$$\Delta q = K \Delta W_f + C_a T_a \Delta W_a \qquad (2\text{-A}13)$$

$$\Delta q_n = \Delta q - \Delta q_{gw} \qquad (2\text{-A}6)$$

$$\Delta q_{re} = \Delta q_n - \Delta q_{sh} \qquad (2\text{-A}7)$$

$$\Delta q_{sh} = \Delta q_s + \Delta q_f + \Delta q_p \qquad (2\text{-C}20)$$

$$\Delta W_g = (1 - R_{hf}) \Delta W_f + \Delta W_a \qquad (2\text{-A}11)$$

$$\Delta T_f = \frac{T_{fo}}{\Delta q_o} \Delta q - \frac{T_{fo}}{W_{go}} \Delta W_g \qquad (2\text{-A}14)$$

$$\Delta q_{gw} = \frac{4 T_{fo}^3 q_{gwo}}{T_o^4 - T_{lo}^4} \Delta T_f - \frac{4 T_{lo}^3 q_{gwo}}{T_{fo}^4 - T_{lo}^4} \Delta T_1 \qquad (2\text{-B}2)$$

$$\Delta q_s = K_1 (\Delta q - \Delta q_{gw}) - K_2 \Delta T_{ms} - K_3 \Delta W_g \qquad (2\text{-C}15)$$

$$\Delta q_f = K_4 (\Delta q - \Delta q_{gw}) + K_5 \Delta T_{ms} - K_6 \Delta T_{mf} - K_7 \Delta W_g \qquad (2\text{-C}17)$$

$$\Delta q_p = K_8 (\Delta q - \Delta q_{gw}) + K_9 \Delta T_{ms} + K_{10} \Delta T_{mf} - K_{11} \Delta T_{mp} - K_{12} \Delta W_g \qquad (2\text{-C}19)$$

b) The temperature of the waterwall tubes is given by Eq. (2-D4). For single layer representation of the tube wall, the variation of the metal temperature is given by

$$\frac{d\Delta T_1}{dt} = \frac{1}{\rho_w c_w V_w} (\Delta q_{gw} - \Delta q_{Dw}) \qquad (3-A1)$$

c) The rate of heat transfer to the fluid inside the waterwall tube is given by Eq. (2-E6)

$$\Delta q_{Dw} = K_{18} \Delta W_D + K_{19} \Delta T_1 - K_{20} \Delta D_D \qquad (3-A2)$$

d) The drum pressure dynamics is given by Eq. (2-F9)

$$\frac{d\Delta D_D}{dt} + \frac{a_1 a_5}{a_1 a_4 - a_2 a_3} \Delta D_D = \frac{a_1 h_e - a_3}{a_1 a_4 - a_2 a_3} \Delta W_{fw} - \frac{a_1 h_{go} - a_3}{a_1 a_4 - a_2 a_3} \Delta W_v$$

$$+ \frac{a_1 \Delta q_{gw}}{a_1 a_4 - a_2 a_3} \qquad (2-F9)$$

e) The dynamics of the superheater tube metal temperature are given by Eq. (2-D4). For single layer representation of the tube wall, the variation of metal temperature of secondary superheater is given by

$$\frac{d\Delta T_{ms}}{dt} = \frac{1}{\rho_{ms} c_{ms} V_{ms}} (\Delta q_s - \Delta q_{Ds}) \qquad (3-A3)$$

The variation of metal temperature of the finishing superheater is given by

$$\frac{d\Delta T_{mf}}{dt} = \frac{1}{\rho_{mf} c_{mf} V_{mf}} (\Delta q_f - \Delta q_{Df}) \qquad (3-A4)$$

The variation of metal temperature of the primary superheater is given by

$$\frac{d\Delta T_{mp}}{dt} = \frac{1}{\rho_{mp} c_{mp} V_{mp}} (\Delta q_p - \Delta q_{Dp}) \tag{3-A5}$$

f) The transfer of heat to the steam in each superheater section is given by Eq. (2-E5). The equations describe the rate of heat flow to steam in the secondary superheater, finishing superheater, and primary superheater are

$$\Delta q_{Ds} = a_{11}\Delta W_s + a_{12}\Delta T_{ms} + a_{13}\Delta D_D - a_{14}\Delta T_{st} \tag{3-A6}$$

$$\Delta q_{Df} = a_{21}\Delta W_f + a_{22}\Delta T_{mf} + a_{23}\Delta D_D - a_{24}\Delta T_{ft} \tag{3-A7}$$

$$\Delta q_{Dp} = a_{31}\Delta W_p + a_{32}\Delta T_{mp} + a_{33}\Delta D_D - a_{34}\Delta T_{pt} \tag{3-A8}$$

where a_{ij} are defined as K_{ij} in Eq. (2-E5) for the respective terms. With the assumption that pressure drops through the super-heaters are negligible, the steam flowing through each superheater may be approximated by

$$\Delta W_p = \Delta W_s = \Delta W_f = \Delta W_T$$

where W_T is the throttle steam flow rate.

g) The dynamics of the outlet steam temperature for each superheater section are determined by Eq. (2-G7). The equations for the respective sections are

$$b_{11}\frac{d\Delta T_{pt}}{dt} + b_{12}\Delta T_{pt} = b_{13}\frac{d\Delta D_D}{dt} - b_{14}\Delta D_D + b_{15}\Delta T_D - b_{16}\Delta W_p + \Delta q_{Dp} \tag{3-A9}$$

$$b_{21}\frac{d\Delta T_{st}}{dt} + b_{22}\Delta T_{st} = b_{23}\frac{d\Delta D_D}{dt} - b_{24}\Delta D_D + b_{25}\Delta T_{pt} - b_{26}\Delta W_s + \Delta q_{Ds} \tag{3-A10}$$

$$b_{31} \frac{d\Delta T_{ft}}{dt} + b_{32}\Delta T_{ft} = b_{33} \frac{d\Delta D_D}{dt} - b_{34}\Delta D_D + b_{35}\Delta T_{st} - b_{36}\Delta W_f + \Delta q_{Df}$$

$$(3\text{-A}11)$$

where b_{ij}'s are defined as b's in Section G of Chapter II.

The intermediate variables Δq_{Dp}, Δq_{Ds}, and Δq_{Df} may be eliminated:

Substituting Eq. (3-A6) into Eq. (3-A10) to obtain

$$b_{21} \frac{d\Delta T_{st}}{dt} + b_{22}\Delta T_{st} = b_{23} \frac{d\Delta D_D}{dt} - b_{24}\Delta D_D + b_{25}\Delta T_{pt} - b_{26}\Delta W_s$$

$$+ a_{11}\Delta W_s + a_{12}\Delta T_{ms} + a_{13}\Delta D_D - a_{14}\Delta T_{st}$$

Rearranging the equation,

$$b_{21} \frac{d\Delta T_{st}}{dt} + (b_{22} + a_{14})\Delta T_{st} = b_{23} \frac{d\Delta D_D}{dt} + (a_{13} - b_{24})\Delta D_D$$

$$- (b_{26} - a_{11})\Delta W_s + a_{12}\Delta T_{ms} + b_{25}\Delta T_{pt} \qquad (3\text{-A}12)$$

Similarly, sustituting Eq. (3-A7) into Eq. (3-A11),

$$b_{31} \frac{d\Delta T_{ft}}{dt} + (b_{32} + a_{24})\Delta T_{ft}$$

$$= b_{33} \frac{d\Delta D_D}{dt} + (a_{23} - b_{34})\Delta D_D - (b_{36} - a_{21})\Delta W_f + a_{22}\Delta T_{mf}$$

$$+ b_{35}\Delta T_{st} \qquad (3\text{-A}13)$$

Substituting Eq. (3-A8) into Eq. (3-A9),

$$b_{11} \frac{d\Delta T_{pt}}{dt} + (b_{12} + a_{34})\Delta T_{pt} = b_{13} \frac{d\Delta D_D}{dt} + (a_{33} - b_{14})\Delta D_D$$

$$- (b_{16} - a_{31})\Delta W_p + a_{32}\Delta T_{mp} + b_{15}\Delta T_D \qquad (3\text{-A}14)$$

h) Equation (2-H4) will be used in the model for throttle steam flow

rate:

$$\Delta W_T = \frac{W_{To}}{A_{To}} \Delta A_T + \frac{W_{To}}{P_{To}} \Delta D_T - \frac{W_{To}}{2T_{To}} \Delta T_T \qquad (3\text{-}A15)$$

With the assumption of negligible pressure drop in the superheater sections, the following approximation can be used

$$\Delta D_D = \Delta D_T$$

$$\Delta T_T = \Delta T_{ft}$$

B. Boiler Model in Frequency Domain

The model differential equations will be transformed to the complex-frequency domain. Then, the boiler model will be established with the transformed equations. The transformation of algebraic equations will not be performed here because they are of the same form as in time domain.

a) The dynamics of waterwall tube metal temperature is given by Eq. (3-A1)

$$\frac{d\Delta T_1}{dt} = \frac{1}{\rho_w c_w V_w} (\Delta q_{gw} - \Delta q_{Dw}) \qquad (3\text{-}A1)$$

The transformed equation is

$$\Delta T_1(s) = \frac{1}{\rho_w c_w V_w} \frac{\Delta q_{gw}(s) - \Delta q_{Dw}(s)}{s} \qquad (3\text{-}B1)$$

b) Drum pressure dynamics

$$\frac{d\Delta D_D}{dt} = - P_4 \Delta D_D + P_5 \Delta W_{fw} - P_6 \Delta W_v + P_7 \Delta q_{Dw} \qquad (2\text{-}F9)$$

where

$$P_4 = \frac{a_1 a_5}{a_1 a_4 - a_2 a_3}$$

$$P_5 = \frac{a_1 h_e - a_3}{a_1 a_4 - a_2 a_3}$$

$$P_6 = \frac{a_1 h_{go} - a_3}{a_1 a_4 - a_2 a_3}$$

$$P_7 = \frac{a_1}{a_1 a_4 - a_2 a_3}$$

and a_1, a_2, a_3, a_4, and a_5 have been defined in Section F of Chapter II. The transformed equation can be written as

$$\Delta D_D(s) = \frac{P_5 \Delta W_{fw}(s) - P_6 \Delta W_v(s) + P_7 \Delta q_{Dw}(s)}{s + P_4} \qquad (3\text{-}B2)$$

c) The basic equations describing the transfer of heat through metal tubes of the superheater sections are the same. For the primary superheater

$$\frac{d \Delta T_{mp}}{dt} = \frac{1}{\rho_{mp} c_{mp} V_{mp}} (\Delta q_p - \Delta q_{Dp}) \qquad (3\text{-}A5)$$

$$\Delta q_{Dp} = a_{31} \Delta W_p + a_{32} \Delta T_{mp} + a_{33} \Delta D_D - a_{34} \Delta T_{pt} \qquad (3\text{-}A8)$$

Combining these two equations:

$$\frac{d \Delta T_{mp}}{dt} = P_8 \Delta q_p - P_9 \Delta D_D + P_{10} \Delta T_{pt} - P_{11} \Delta W_p - P_{12} \Delta T_{mp}$$

where

$$P_8 = \frac{1}{\rho_{mp} c_{mp} V_{mp}}$$

$$P_9 = \frac{a_{33}}{\rho_{mp} c_{mp} V_{mp}}$$

$$P_{10} = \frac{a_{34}}{\rho_{mp} c_{mp} V_{mp}}$$

$$P_{11} = \frac{a_{31}}{\rho_{mp} c_{mp} V_{mp}}$$

$$P_{12} = \frac{a_{32}}{\rho_{mp} c_{mp} V_{mp}}$$

The transformed equation for the primary superheater metal temperature can be written as

$$\Delta T_{mp}(s) = \frac{P_8 \Delta q_p(s) - P_9 \Delta D_D(s) + P_{10} \Delta T_{pt}(s) - P_{11} \Delta W_p(s)}{s + P_{12}} \quad (3\text{-}B3)$$

For the same reason, the equations for the secondary superheater temperature and the finishing superheater temperature are Eqs. (3-B4) and (3-B5), respectively.

$$\Delta T_{ms}(s) = \frac{P_{13} \Delta q_s(s) - P_{14} \Delta D_D(s) + P_{15} \Delta T_{st}(s) - P_{16} \Delta W_s(s)}{s + P_{17}} \quad (3\text{-}B4)$$

$$\Delta T_{mf}(s) = \frac{P_{18} \Delta q_f(s) - P_{19} \Delta D_D(s) + P_{20} \Delta T_{ft}(s) - P_{21} \Delta W_f(s)}{s + P_{22}} \quad (3\text{-}B5)$$

where

$$P_{13} = \frac{1}{\rho_{ms} c_{ms} V_{ms}}$$

$$P_{14} = \frac{a_{13}}{\rho_{ms} c_{ms} V_{ms}}$$

$$P_{15} = \frac{a_{14}}{\rho_{ms} c_{ms} V_{ms}}$$

$$P_{16} = \frac{a_{11}}{\rho_{ms} c_{ms} V_{ms}}$$

$$P_{17} = \frac{a_{12}}{\rho_{ms} c_{ms} V_{ms}}$$

$$P_{18} = \frac{1}{\rho_{mf} c_{mf} V_{mf}}$$

$$P_{19} = \frac{a_{23}}{\rho_{mf} c_{mf} V_{mf}}$$

$$P_{20} = \frac{a_{24}}{\rho_{mf} c_{mf} V_{mf}}$$

$$P_{21} = \frac{a_{21}}{\rho_{mf} c_{mf} V_{mf}}$$

$$P_{22} = \frac{a_{22}}{\rho_{mf} c_{mf} V_{mf}}$$

d) The transformed equations for outlet steam temperature of the super-heaters are obtained from Eqs. (3-A12), (3-A13), and (3-A14), respectively.

$$\Delta T_{st}(s) = \frac{P_{23}(s + P_{24})\Delta D_D(s) + P_{25}\Delta T_{Dt} - P_{26}\Delta W_s + P_{27}\Delta T_{ms}}{s + P_{28}} \quad (3\text{-}B6)$$

$$\Delta T_{dt}(s) = \frac{P_{29}(s + P_{30})\Delta D_D(s) + P_{31}\Delta T_{st} - P_{32}\Delta W_f + P_{33}\Delta T_{mf}}{s + P_{34}} \quad (3\text{-}B7)$$

$$\Delta T_{pt}(s) = \frac{P_{35}(s + P_{36})\Delta D_D(s) + P_{37}\Delta T_D - P_{38}\Delta W_p + P_{39}\Delta T_{mp}}{s + P_{40}} \quad (3\text{-}B8)$$

where

$$P_{23} = \frac{b_{23}}{b_{21}}$$

$$P_{24} = \frac{a_{13} - b_{24}}{b_{23}}$$

$$P_{25} = \frac{b_{25}}{b_{21}}$$

$$P_{26} = \frac{b_{26} - a_{11}}{b_{21}}$$

$$P_{27} = \frac{a_{12}}{b_{21}}$$

$$P_{28} = \frac{b_{22} + a_{14}}{b_{21}}$$

$$P_{29} = \frac{b_{33}}{b_{31}}$$

$$P_{30} = \frac{a_{23} - b_{34}}{b_{33}}$$

$$P_{31} = \frac{b_{35}}{b_{31}}$$

$$P_{32} = \frac{b_{36} - a_{21}}{b_{31}}$$

$$P_{33} = \frac{a_{22}}{b_{31}}$$

$$P_{34} = \frac{b_{32} + a_{24}}{b_{31}}$$

$$P_{35} = \frac{b_{13}}{b_{11}}$$

$$P_{36} = \frac{a_{33} - b_{14}}{b_{13}}$$

$$P_{37} = \frac{b_{15}}{b_{11}}$$

$$P_{38} = \frac{b_{16} - a_{31}}{b_{11}}$$

$$P_{39} = \frac{a_{32}}{b_{11}}$$

$$P_{40} = \frac{b_{12} + a_{34}}{b_{11}}$$

The model block diagram is shown in Fig. 5. The constants indicated in Fig. 5 are defined below, where the constants on the right hand side have been defined in the derivation of the equations.

$$L_1 = K_f + C_f T_{f1} - C_{ash} T_{ash} R_{hf}$$

$$L_2 = C_a T_a$$

$$L_3 = 1$$

$$L_4 = 1 - R_{hf}$$

$$L_5 = 4q_{gwo} T_{fo}^3 / (T_{fo}^4 - T_{lo}^4)$$

$$L_6 = T_{fo}/q_o$$

$$L_7 = T_{fo}/W_{go}$$

$$L_8 = 4T_{lo}^3 / (T_{fo}^4 - T_{lo}^4)$$

$$L_9 = 1/P_w c_w V_w$$

$$L_{10} = K_{16} q_{Do} / T_{lo}$$

$$L_{11} = K_{15} q_{Do} / W_{Do}$$

$$L_{12} = K_{17} q_{Do} / D_{Do}$$

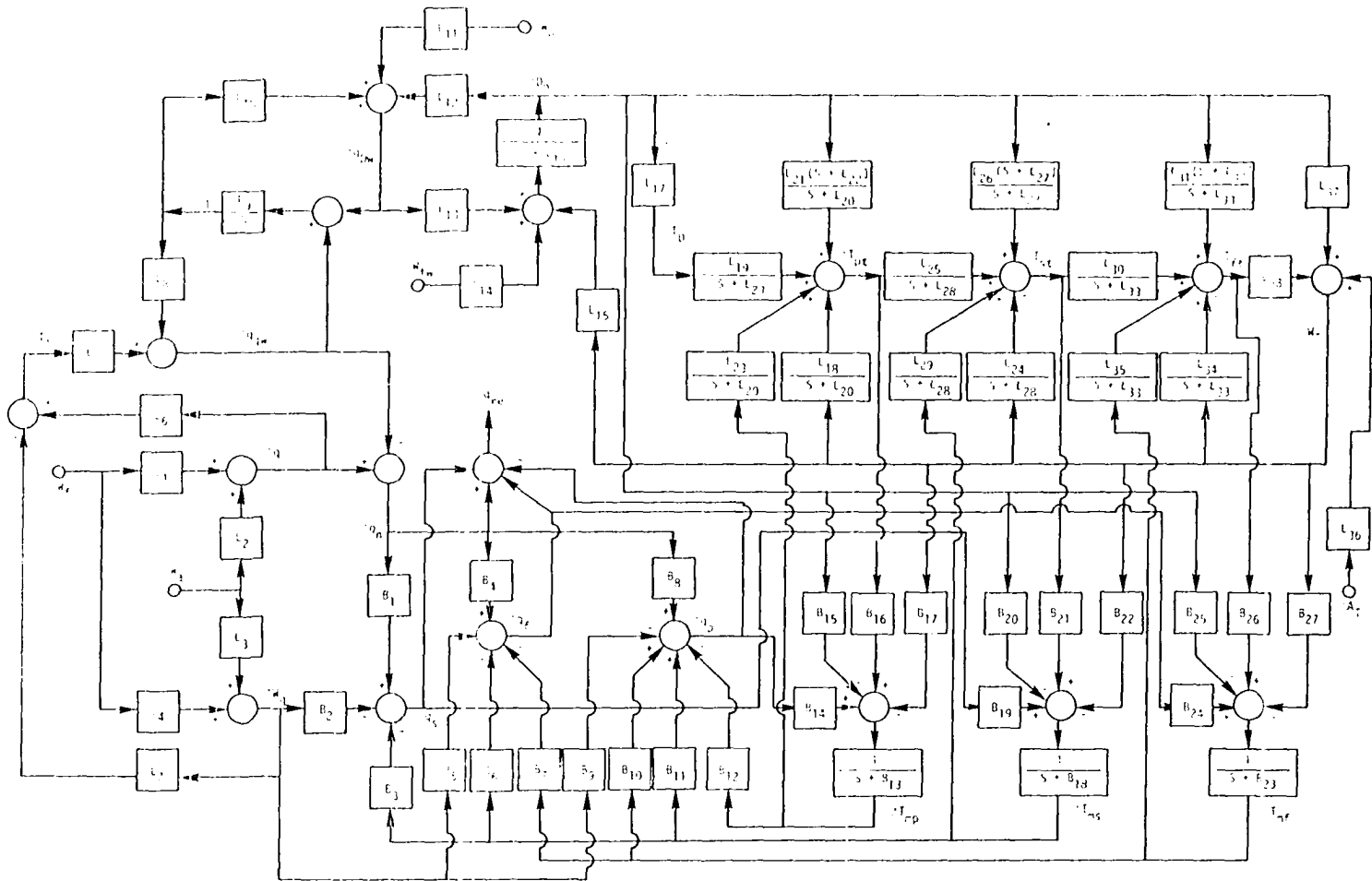$$L_{13} = a_1 / (a_1 a_4 - a_2 a_3) = P_7$$

Fig. 5. A model for a drum type boiler.

$$L_{14} = (a_1 h_e - a_3)/(a_1 a_4 - a_2 a_3) = P_5$$

$$L_{15} = a_1 a_5/(a_1 a_4 - a_2 a_3) = P_4$$

$$L_{16} = (a_1 h_{go} - a_3)/(a_1 a_4 - a_2 a_3) = P_6$$

$$L_{17} = T_{Do} v_{fgo}/h_{fgo}$$

$$L_{18} = (b_{16} - a_{31})/b_{11} = P_{38}$$

$$L_{19} = b_{15}/b_{11} = P_{37}$$

$$L_{20} = (b_{12} + a_{34})/b_{11} = P_{40}$$

$$L_{21} = b_{13}/b_{11} = P_{35}$$

$$L_{22} = (a_{33} - b_{14})/b_{13} = P_{36}$$

$$L_{23} = a_{32}/b_{11} = P_{39}$$

$$L_{24} = (b_{26} - a_{11})/b_{11}$$

$$L_{25} = b_{25}/b_{21} = P_{25}$$

$$L_{26} = b_{23}/b_{21} = P_{23}$$

$$L_{27} = (a_{13} - b_{24})/b_{23} = P_{24}$$

$$L_{28} = (b_{22} + a_{14})/b_{21} = P_{28}$$

$$L_{29} = a_{12}/b_{21} = P_{27}$$

$$L_{30} = b_{35}/b_{31} - P_{31}$$

$$L_{31} = b_{33}/b_{31} = P_{29}$$

$$L_{32} = (a_{23} - b_{34})/b_{33} = P_{30}$$

$$L_{33} = (b_{33} + a_{24})/b_{31} = P_{31}$$

$$L_{34} = (b_{36} - a_{21})/b_{31} = P_{32}$$

$$L_{35} = a_{22}/b_{31} = P_{33}$$

$$L_{36} = W_{To}/A_{To}$$

$$L_{37} = W_{To}/P_{To}$$

$$L_{38} = W_{To}/2T_{To}$$

$$B_1 = K_1$$

$$B_2 = K_3$$

$$B_3 = K_2$$

$$B_4 = K_4$$

$$B_5 = K_7$$

$$B_6 = K_5$$

$$B_7 = K_6$$

$$B_8 = K_8$$

$$B_9 = K_{12}$$

$$B_{10} = K_{10}$$

$$B_{11} = K_9$$

$$\bar{B}_{12} = \bar{K}_{11}$$

$$B_{13} = a_{32}/\rho_{mp} c_{mp} \bar{V}_{mp} = P_{12}$$

$$B_{14} = 1/\rho_{mp} c_{mp} V_{mp} = P_8$$

$$B_{15} = a_{33}/\rho_{mp} c_{mp} V_{mp} = P_9$$

$$B_{16} = a_{34}/\rho_{mp}c_{mp}V_{mp} = P_{10}$$

$$B_{17} = a_{31}/\rho_{mp}c_{mp}V_{mp} = P_{11}$$

$$B_{18} = P_{17}$$

$$B_{19} = P_{13}$$

$$B_{20} = P_{14}$$

$$B_{21} = P_{15}$$

$$B_{22} = P_{16}$$

$$B_{23} = P_{22}$$

$$B_{24} = P_{18}$$

$$B_{25} = P_{19}$$

$$B_{26} = P_{20}$$

$$B_{27} = P_{21}$$

The model developed here is through detailed analyses of boiler

processes. A comparison of some features of physical models published in

the 1970's (15,16,18,19,27) with the present model is given below.

a) Type of boiler considered:

Kwan and Anderson (15): drum-type, natural circulation.

Shang(27): once-through boiler.

Kwatny et al. (16) and McDonald and Kwatny (19): drum-type, forced

circulation.

McDonald (18): drum-type, forced circulation.

Present: drum-type, forced circulation.

b) Heat transfer through waterwall tubes:

Kwan and Anderson: thermal inertia of tube metal is considered.

Shang: thermal inertia of tube metal is considered.

McDonald and Kwatny: thermal inertia of tube metal is not considered.

McDonald: thermal inertia of tube metal is considered.

Present: thermal inertia of tube metal is considered.

c) Fluid flow in waterwall tubes:

Kwan and Anderson: fluid flow rate is considered constant.

Shang: fluid flow rate is controlled.

McDonald and Kwatny: fluid flow rate is considered constant.

McDonald: fluid flow rate is considered constant.

Present: fluid flow rate is controlled.

d) Drum pressure dynamics:

Kwan and Anderson: drum pressure is not an explicit state variable, but may be solved with a combination of model algebraic equations and integral equations.

Shang: (no drum)

McDonald and Kwatny: neither drum pressure nor steam temperature in drum is an output of model equations.

McDonald: drum pressure is an integral function of the rate of heat transfer to circulation fluid and the flow rate of steam out from the drum.

Present: drum pressure is a function of the rate of heat transfer to circulation fluid, the steam flow rate, and the feedwater flow rate; involving a delay.

e) Superheater dynamics:

    Kwan and Anderson: thermal inertia of tube metal is considered.

    Shang: thermal inertia of tube metal is considered.

    McDonald and Kwatny: thermal inertia of tube metal is not considered.

    McDonald: dynamics of superheaters are not considered; throttle

temperature is considered constant.

    Present: thermal inertia of tube metal is considered.

C. A Simplified Boiler Model

A simplified model may be not accurate enough to represent a boiler

for the long periods required for control. They are useful in predicting

the variation of boiler variables for shorter periods. One simplification

is to treat the three superheaters as one section; then the model is reduced

to that shown in Fig. 6.

A power plant boiler is usually equipped with an independent feedwater

control loop such that the feedwater flow rate is equal to the steam flow

rate. If this control is perfect and is considered as part of the boiler

system, then

$$\Delta W_{fw} = \Delta W_T$$

and the model terminals ΔW_{fw} and ΔW_T can be connected together. Also, if

the perturbation is small so that W_D may be kept as constant, then $\Delta W_D = 0$.

Further simplification of the model can be done on the gas side. If

fuel and air flow control is perfect such that fuel flow rate is proportional

to air flow rate, then

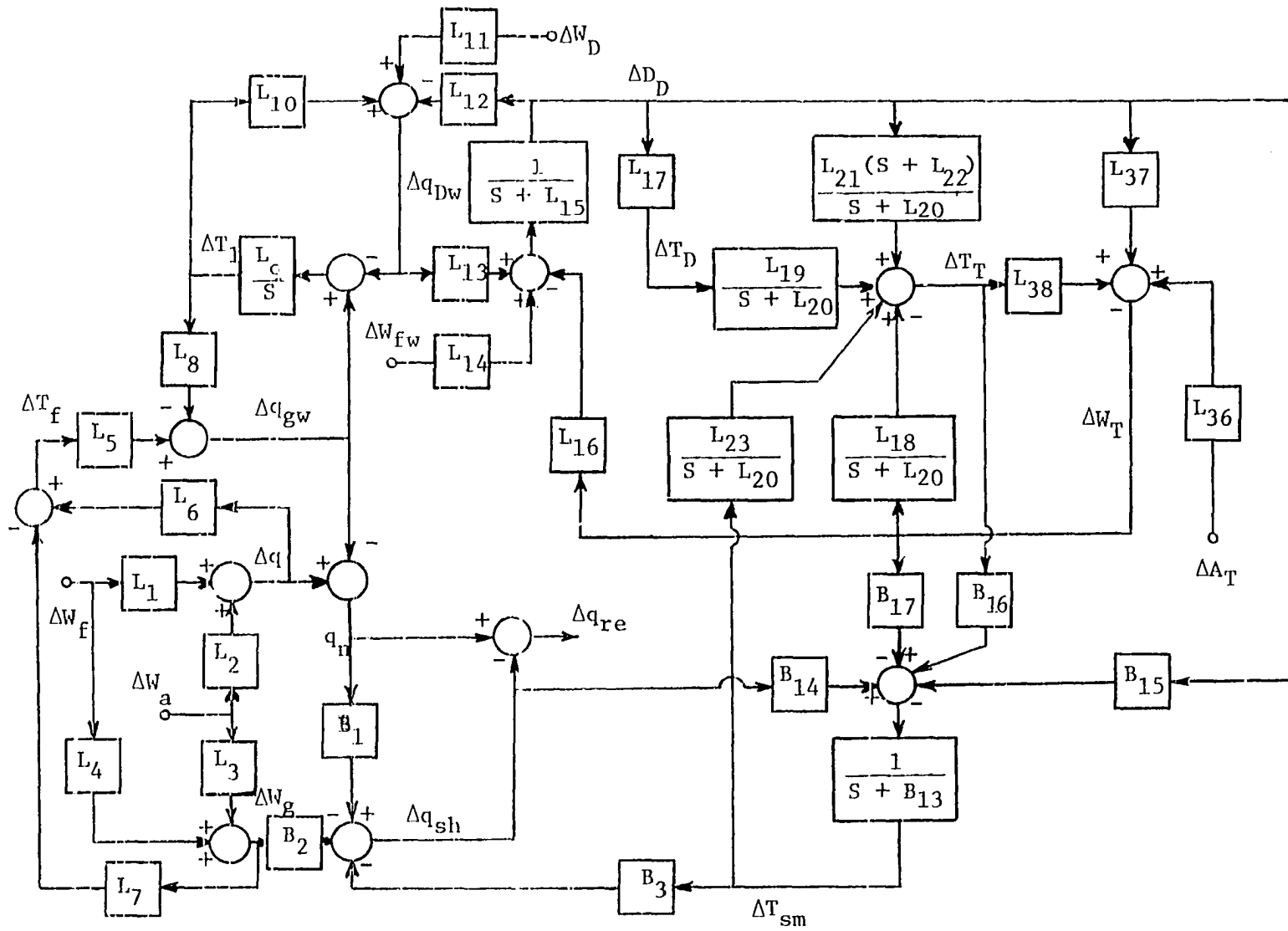$$\Delta W_a = R_{fd} \Delta W_f$$

Fig. 6. Diagram of a simplified model.

and Eq. (2-A13) becomes

$$\Delta q = (K - C_a T_a R_{fa}) \Delta W_f \qquad (3-C1)$$

where R_{fa} is air to fuel ratio. Also, during steady-state conditions, the heat transfer to each boiler section is a constant fraction of the total heat production in the furnace. If the proportionality is not changed for small boiler perturbations, then

$$\Delta q_{gw} = R_{gw} \Delta q = R_{gw}(K - C_a T_a R_{fa}) \Delta W_f \overset{\Delta}{=} F_1 \Delta W_f \qquad (3-C2)$$

$$\Delta q_{sh} = R_{sh} \Delta q = R_{sh}(K - C_a T_a R_{fa}) \Delta W_f \overset{\Delta}{=} F_2 \Delta W_f \qquad (3-C3)$$

where

$$R_{gw} < 1$$

and

$$R_{sh} < 1$$

are constants.

With these relations applied, the boiler model can be reduced to that shown in Fig. 7, where the two blocks involving L_{19} and L_{22} are also combined. Also, the loops enclosed in the dashed box can be reduced as given in Appendix C. Then the model becomes that shown in Fig. 8, where

$$F_3 = L_{21} L_{22} + L_{17} L_{19} \quad .$$

The lower summing function in Fig. 8 can be eliminated. Fig. 9 shows the direct result with elimination of this function.
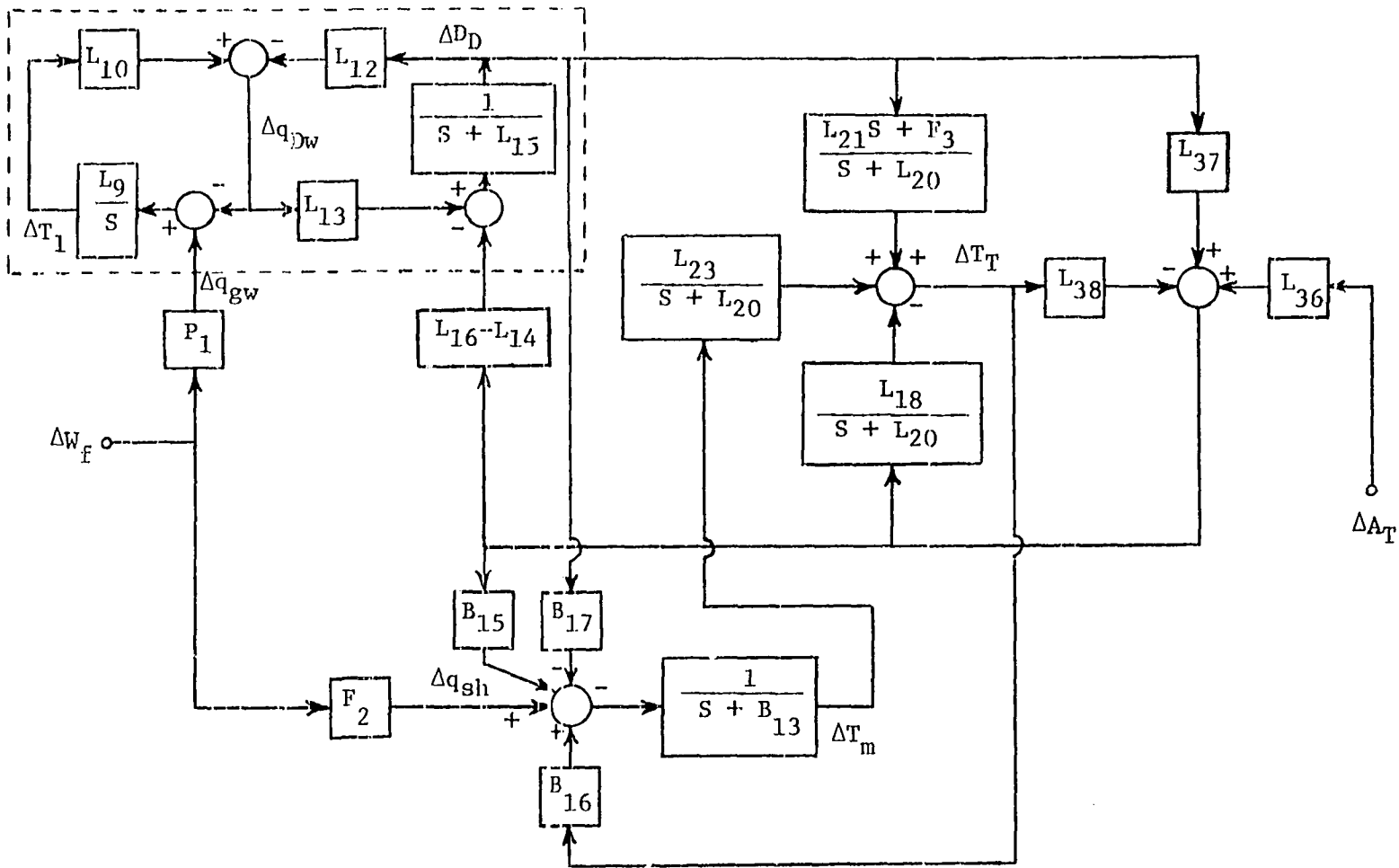
Fig. 7. Boiler model with simplification on gas processes.
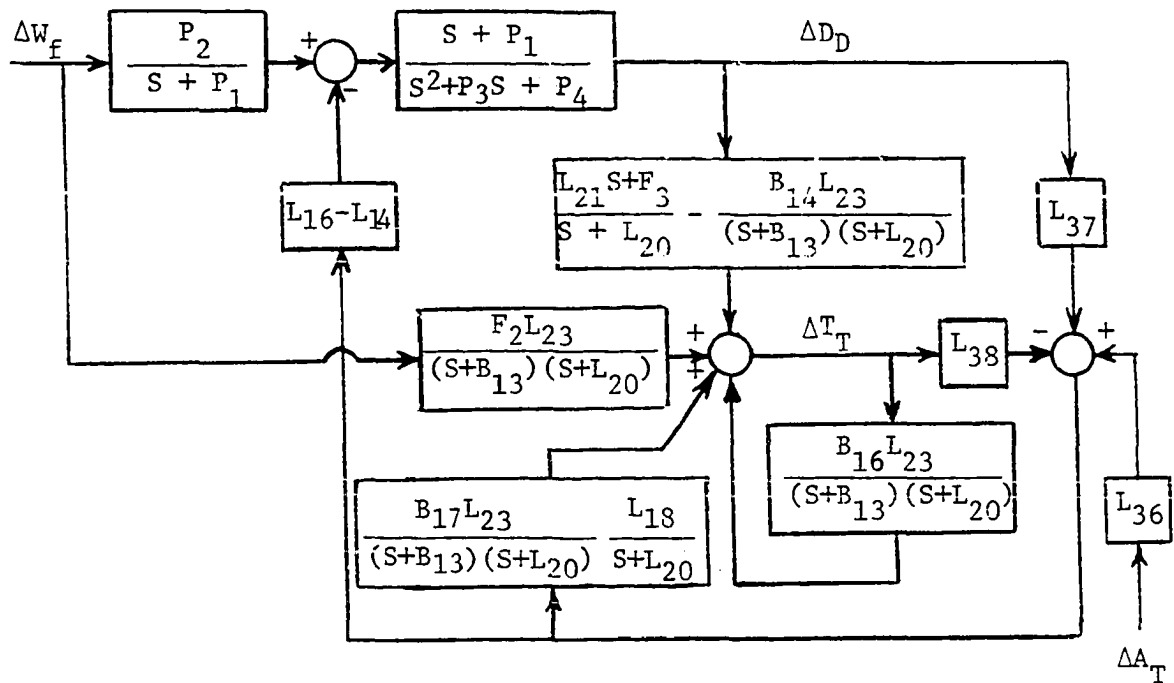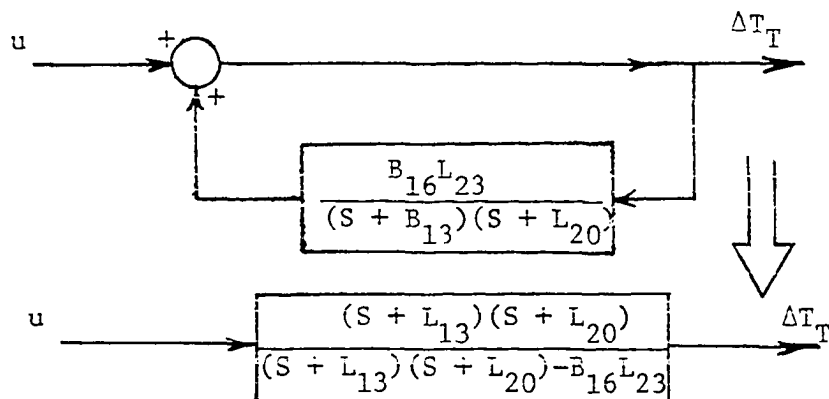
Fig. 8. A reduced boiler model.

Fig. 9. Boiler model with lower summing junction
in Fig. 8 eliminated.

The loop with ΔT_T can be reduced, as shown below.

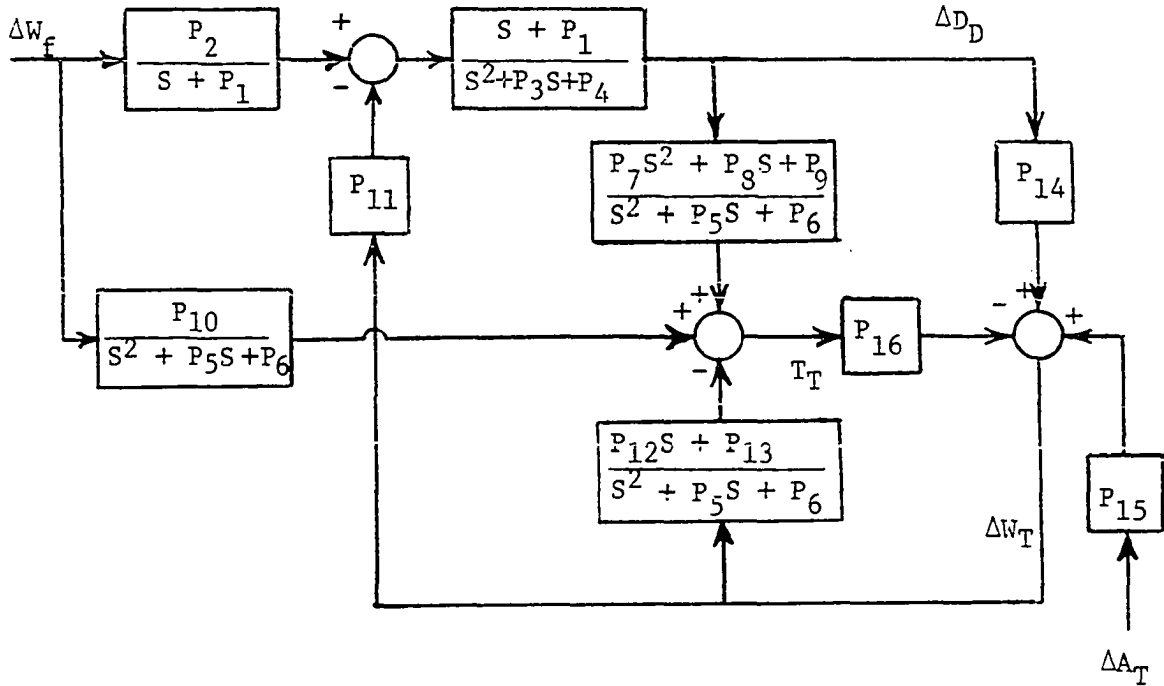The model is finally reduced to that shown in Fig. 10.

Fig. 10. The final simplified boiler model.

where

$$P_5 = B_{13} + L_{20}$$

$$P_6 = B_{13}L_{20} - B_{16}L_{23}$$

$$P_7 = L_{21}$$

$$P_8 = L_{21}B_{13} + F_3 = L_{21}L_{13} + L_{21}L_{22} + L_{17}L_{19}$$

$$P_9 = B_{13}F_3 - B_{15}L_{23} + B_{13}(L_{21}L_{22} + L_{17}L_{19}) - B_{15}L_{23}$$

$$P_{10} = F_2L_{23}$$

$$P_{11} = L_{16} - L_{14}$$

$$P_{12} = L_{18}$$

$$P_{13} = B_{13}L_{18} + B_{17}L_{23}$$

This is a low order model developed on physical principles. The model inputs are

$$\Delta W_f = \text{fuel flow rate and}$$

$$\Delta A_T = \text{control valve area,}$$

and the model outputs are

$$\Delta D_D = \text{drum pressure,}$$

$$\Delta T_T = \text{throttle temperature, and}$$

$$\Delta W_T = \text{throttle steam flow rate.}$$

IV. DATA MEASUREMENT

Data of boiler dynamic responses are required for computation of model

parameter constants. These data must be measured from a physical boiler at

the points corresponding to the inputs and outputs of the model. Since the

boiler model does not include transfer functions of control equipment and

measurement equipment, the data recorded should be the direct reproduction

of actual signals. In case distortion caused by measurement equipment is

significant and only control signals are available, the transfer functions

of the control and measurement equipment must be connected to the corre-

sponding terminals of the boiler model. The model including the transfer

functions of equipment should be used with the data for computing the

parameter constants. This may not increase the number of unknown model

parameters, since the transfer functions of the measurement equipment and

the control equipment and their constants may be determined beforehand.

In making experimental tests, it must be carefully observed that the

boiler system is in steady-state before the inputs are perturbed and the

input and the output signals are recorded. It is desirable to obtain sets

of boiler response data with each individual input perturbed as well as

with a combination of several inputs perturbed simultaneously so that the

sensitivity of individual inputs to the boiler dynamics and their combined

effects on the system performances can be understood. The data set for

parameter identification computation should include the following infor-

mation:

Data for model inputs

     1) fuel flow rate,

     2) air flow rate,

     3) control valve area,

     4) feedwater flow rate, and

     5) flow rate of circulation fluid.

Data for model outputs

     1) throttle steam flow rate,

     2) throttle steam temperature,

     3) drum pressure, and

     4) rate of heat flow to reheater and economizer sections.

ASME Performance Test Code and ASME Power Test Code may be followed to obtain the test data.

Fuel flow rate, air flow rate, control valve area, feedwater flow rate, throttle steam flow rate, and drum pressure generally can be measured with the equipment normally installed in the plant. The rate of heat flow to the reheater and economizer can not be measured directly. It has to be computed from the data measured for

     1) temperature of the flue gas flowing into the reheater section,

     2) temperature of the flue gas flowing into the economizer section,

     3) mass flow rate of the flue gas into the reheater sections, and

     4) mass flow rate of the flue gas into the economizer section.

The rate of heat flow into the respective sections then can be computed. with Eqs. (2-A8) and (2-A9). The instrument for measuring these quantities is usually not installed in the plants.

The flow rate of circulation fluid usually can not be measured. Instruments have to be installed to make the test. It will be convenient if the transfer function relating the circulation fluid flow rate and the signal of the circulation pump driving motor input is obtained before the test. In this case the transfer function can be connected to the boiler model and the motor input becomes an input to the boiler, as shown in Fig. 11, where ΔM_I is the motor input.
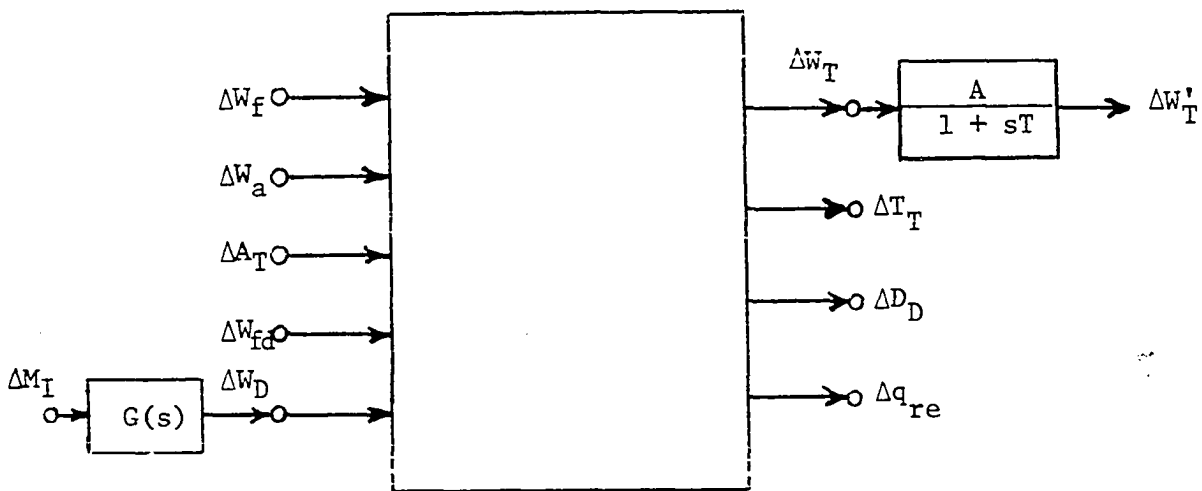

Fig. 11. Inputs and outputs of boiler model.

The throttle steam flow rate is proportional to the turbine first stage shell pressure corrected with the throttle temperature (2,26). A common practice is that the first stage pressure is measured and corrected to produce the data of steam flow rate. However, there is a time lag between the steam flow through the throttle valve and the detected variation of first stage pressure. The lag is mainly due to the existence of the

steam chest and the connecting pipe between the control valves and the

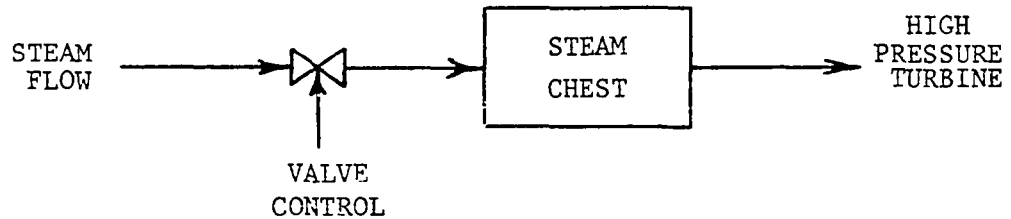turbine first-stage shell, as shown in Fig. 12.

Fig. 12. Time lag exists between control valve
          steam flow and turbine first stage pressure.

This time lag appearing in the recorded data influences the results of

identification computation if it is not properly treated. It has been

suggested that this lag can be represented by a first order delay (13). A

first order transfer function can be connected to the output ΔW_T to repre-

sent this lag, as shown in Fig. 11, where ΔW_T is the actual steam flow rate

and $\Delta W_T'$ is the measured steam flow rate.

Power plants are generally equipped with automatic controls such that

the feedwater flow rate is controlled to equal the rate of steam flow out of

the drum, and the air flow rate is controlled proportional to the fuel flow

rate. If the controls are perfect, the information about feedwater flow

rate and air flow rate become unnecessary. However, it is more desirable

to have this information measured so that the actual situation is understood.

The test data should be recorded for a period of at least twenty

minutes, as it sometimes takes about twenty minutes for a major thermal

transient to subside. Since the superheater controls are not included in the boiler model, superheater spray should be kept off during the experiment.

The boiler dynamic data should be recorded on analog magnetic tapes during the test so that they can be digitized later for digital computer application. It is trivial to mention that an experiment daily should be kept on file for the test. However, a portion or all of the data recorded will lose their value if the following information is not available:

a) the environment pressure during the experiment must be known,

b) the scale factors for all the records must be known,

c) base lines which show steady-state levels of boiler variables must be shown clearly in all records,

d) the physical values which the base lines represent must be known,

e) the polarity of the recording voltage must be known,

f) the formal recording on all data must start at the same time, and

g) the location and length of each set of data on the tapes must be known.

Since a boiler is a large system and it is not always available for experiments, care must be taken to obtain this information in doing the experiment.

V. CONCLUSIONS

A linear mathematical model for a thermal power plant drum type boiler has been developed. This model is intended for multivariable control studies on the boiler. It has included the pressure and temperature aspects of a boiler system. The dynamics of these pressure and temperature variables are important to control studies. Unlike the boiler models developed previously and discussed in Chapter I, in which some boiler controls are considered as a portion of the boiler, the boiler model developed here does not include controls. Some simplified boiler models are also presented. For these models, some external controls are included in the models so that assumptions can be made with the feedwater flow rate and the air flow rate. The simplified model may not be suitable for boiler control studies, but they may be used for an initial study of boiler dynamics.

The model is presented in closed compact form in the complex frequency domain, showing clearly the relationships among the boiler variables. Care has been taken in the model development that the boiler processes are described by a suitable set of boiler variables which not only represent the important boiler properties but also provide good relationships between the boiler components. Since the model is developed from physical principles, it can be used to represent boilers of the same configuration.

To compute the model constants for a particular boiler, dynamic input-output response data measured from experimental tests on the boiler are required to fit the model. The parameter identification technique is available for computation of the model constants. Initial estimates of the parameter constants, which are required for the parameter identification

program, can be calculated from boiler design data. The relations between the model parameters and the physical quantities of the boiler have been defined in the model development. The computed model parameter values are the effective dynamic constants of the boiler processes which give the corresponding input-output relations.

The dynamic response data for the inputs and the outputs of the boiler model are necessary for computation of the model constants. It is desirable if data are available for the other boiler variables so that the parameter computation can also be done on a partitioned model. In the measurement of the transient pressure and the transient temperature, care must be taken in visualizing the possible time delay appearing in the recorded signal which is usually caused by the sensing the measurement system. These delays should be properly corrected for application to parameter identification computation.

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VII. ACKNOWLEDGMENTS

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APPENDIX A.   TRANSFER FUNCTION FOR WATERWALL TUBE WALL
DIVIDED INTO THREE LAYERS

Equation (2-D4) can be written as

$$\frac{d}{dt}\begin{bmatrix} \Delta T_1 \\ \Delta T_2 \\ \Delta T_3 \end{bmatrix} = \begin{bmatrix} -\dfrac{A_1 h_1}{\rho_1 c_1 V_1} & \dfrac{A_2 h_2}{\rho_1 c_1 V_1} & 0 \\ \dfrac{A_1 h_1}{\rho_2 c_2 V_2} & -\dfrac{(A_1 h_1 + A_2 h_2)}{\rho_2 c_2 V_2} & \dfrac{A_2 h_2}{\rho_2 c_2 V_2} \\ 0 & \dfrac{A_2 h_2}{\rho_3 c_3 V_3} & \dfrac{-A_3 h_3}{\rho_3 c_3 V_3} \end{bmatrix}\begin{bmatrix} \Delta T_1 \\ \Delta T_2 \\ \Delta T_3 \end{bmatrix}$$

$$+ \begin{bmatrix} \dfrac{1}{\rho_1 c_1 V_1} & 0 \\ 0 & 0 \\ 0 & \dfrac{-1}{\rho_3 c_3 V_3} \end{bmatrix}\begin{bmatrix} \Delta q_{gw} \\ \Delta q_{Dw} \end{bmatrix} \tag{A-1}$$

Equation (2-E6) is

$$\Delta q_{Dw} = K_{19}\Delta T_3 + K_{18}\Delta W_D - K_{20}\Delta D_D \tag{A-2}$$

Combining Eqs. (A-1) and (A-2), eliminating Δq_{Dw}

$$\frac{d}{dt}\begin{bmatrix} \Delta T_1 \\ \Delta T_2 \\ \Delta T_3 \end{bmatrix} = \begin{bmatrix} \dfrac{-A_1 h_1}{\rho_1 c_1 V_1} & \dfrac{A_2 h_2}{\rho_1 c_1 V_1} & 0 \\ \dfrac{A_1 h_1}{\rho_2 c_2 V_2} & \dfrac{-(A_1 h_1 + A_2 h_1)}{\rho_2 c_2 V_2} & \dfrac{A_2 h_2}{\rho_2 c_2 V_2} \\ 0 & \dfrac{A_2 h_2}{\rho_3 c_3 V_3} & \dfrac{-(A_3 h_3 + K_{19})}{\rho_3 c_3 V_3} \end{bmatrix}\begin{bmatrix} \Delta T_3 \\ \Delta T_3 \\ \Delta T_3 \end{bmatrix}$$

$$+ \begin{bmatrix} \dfrac{1}{\rho_1 c_1 V_1} & 0 & 0 \\[2ex] 0 & 0 & 0 \\[2ex] 0 & \dfrac{-K_{18}}{\rho_3 c_3 V_3} & \dfrac{K_{20}}{\rho_3 c_3 V_3} \end{bmatrix} \begin{bmatrix} \Delta q_{gw} \\[2ex] \Delta W_D \\[2ex] \Delta D_D \end{bmatrix} \qquad (A\text{-}3)$$

where Δq_{gw}, ΔW_D, and ΔD_D are the inputs to this subsystem, and Eq. (A-2) is the output equation.

To express this subsystem in the form of transfer functions, the state-space equations can be transformed to

$$\Delta T_1(s) = \frac{1}{s + \dfrac{A_1 h_1}{\rho_1 c_1 V_1}} \left[\frac{A_2 h_2}{\rho_1 c_1 V_1} \Delta T_2(s) + \frac{1}{\rho_1 c_1 V_1} \Delta q_{gw}(s) \right] \qquad (A\text{-}4)$$

$$\Delta T_2(s) = \frac{1}{s + \dfrac{A_1 h_1 + A_2 h_2}{\rho_2 c_2 V_2}} \left[\frac{A_1 h_1}{\rho_2 c_2 V_2} \Delta T_1(s) + \frac{A_2 h_2}{\rho_2 c_2 V_2} \Delta T_3(s) \right] \qquad (A\text{-}5)$$

$$\Delta T_3(s) = \frac{1}{s + \dfrac{A_3 h_3 + K_{19}}{\rho_3 c_3 V_3}} \left[\frac{A_2 h_2}{\rho_3 c_3 V_3} \Delta T_2(s) - \frac{K_{18}}{\rho_3 c_3 V_3} \Delta W_D(s) \right.$$

$$\left. \frac{K_{20}}{\rho_3 c_3 V_3} \Delta D_D(s) \right] \qquad (A\text{-}6)$$

Substitute Eq. (A-4) into Eq. (A-5) and rearrange,

$$\Delta T_2(s) = \frac{\dfrac{1}{\rho_2 c_2 V_2}}{s^2 + \left(\dfrac{A_1 h_1}{\rho_1 c_1 V_1} + \dfrac{A_1 h_1 + A_2 h_2}{\rho_2 c_2 V_2} \right) s + \dfrac{A_1^2 h_1^2}{\rho_1 c_1 V_1 \rho_2 c_2 V_2}} \qquad \times$$

$$\left[\frac{A_2 h_2 \left(s + \frac{A_1 h_1 + A_2 h_2}{\rho_1 c_1 V_1 \, _2 c_2 V_2} \right) \left(s + \frac{A_1 h_1}{\rho_1 c_1 V_1} \right)}{s + \frac{A_1 h_1 + A_2 h_2}{\rho_2 c_2 V_2}} \Delta T_3(s) + \frac{A_1 h_1}{\rho_1 c_1 V_1} \Delta q_{gw}(s) \right]$$

(A-7)

Substitute Eq. (A-7) into Eq. (A-6) and rearrange,

$$T_3(s) = \frac{1}{s^4 + P_1 s^3 + P_2 s^2 + P_3 s + P_4} \left[P_5 (s + P_6) \Delta q_{gw}(s) \right.$$

$$\left. -(s^3 + P_7 s^2 + P_8 s + P_9)(P_{10} \Delta W_D(s) - P_{11} \Delta D_D(s)) \right]$$

(A-8)

where

$$P_1 = \frac{A_3 h_3 + K_{19}}{\rho_3 c_3 V_3} + 2 \frac{A_1 h_1 + A_2 h_2}{\rho_2 c_2 V_2} + \frac{A_1 h_1}{\rho_1 c_1 V_1}$$

$$P_2 = \frac{(A_3 h_3 + K_{19})(A_1 h_1 + A_2 h_2)}{\rho_2 c_2 V \rho_3 c_3 V_3} + \left(\frac{A_1 h_1}{\rho_1 c_1 V_1} + \frac{A_1 h_1 + A_2 h_2}{\rho_2 c_2 V_2} \right) \quad \times$$

$$\left(\frac{A_3 h_3 + K_{19}}{\rho_3 c_3 V_3} + \frac{A_1 h_1 + A_2 h_2}{\rho_2 c_2 V_2} \right) + \frac{A_1^2 h_1^2}{\rho_1 c_1 V_1 \rho_2 c_2 V_2} - \frac{A_2^2 h_2^2}{\rho_2 c_2 V_2 \rho_3 c_3 V_3}$$

$$P_3 = \frac{A_1^2 h_1^2 (A_3 h_3 + K_{19})}{\rho_1 c_1 V_1 \rho_2 c_2 V_2 \rho_3 c_3 V_3} + \left(\frac{A_1 h_1}{\rho_1 c_1 V_1} + \frac{A_1 h_1 + A_2 h_2}{\rho_2 c_2 V_2} \right) \quad \times$$

$$\frac{(A_3h_3 + K_{19})(A_1h_1 + A_2h_2)}{\rho_2c_2V_2\rho_3c_3V_3} + \frac{A_1^2h_1^2(A_1h_1 + A_2h_2)}{\rho_1c_1V_1(\rho_2c_2V_2)^2}$$

$$- \frac{A_2^2h_2^2}{\rho_2c_2V_2\rho_3c_3V_3}\left(\frac{A_1h_1 + A_2h_2}{\rho_1c_1V_1\rho_2c_2V_2} + \frac{A_1h_1}{\rho_1c_1V_1}\right)$$

$$P_4 = \frac{A_1^2h_1^2(A_3h_3 + K_{19})(A_1h_1 + A_2h_2)}{\rho_1c_1V_1(\rho_2c_2V_2)^2\rho_3c_3V_3} - \frac{A_1h_1(A_2h_2)^2(A_1h_1 + A_2h_2}{(\rho_1c_1V_1\rho_2c_2V_2)^2\rho_3c_3V_3}$$

$$P_5 = \frac{A_1h_1A_2h_2}{\rho_1c_1V_1\rho_2c_2V_2\rho_3c_3V_3}$$

$$P_6 = \frac{A_1h_1 + A_2h_2}{\rho_2c_2V_2}$$

$$P_7 = 2\frac{A_1h_1 + A_2h_2}{\rho_2c_2V_2} + \frac{A_1h_1}{\rho_1c_1V_1}$$

$$P_8 = \left(\frac{A_1h_1}{\rho_1c_1V_1} + \frac{A_1h_1 + A_2h_2}{\rho_2c_2V_2}\right)\left(\frac{A_1h_1 + A_2h_2}{\rho_2c_2V_2}\right) + \frac{A_1^2h_1^2}{\rho_1c_1V_1\rho_2c_2V_2}$$

$$P_9 = \frac{A_1^2h_1^2(A_1h_1 + A_2h_2)}{\rho_1c_1V_1(\rho_2c_2V_2)^2}$$

$$P_{10} = \frac{K_{18}A_1h_1A_2h_2}{\rho_1c_1V_1\rho_2c_2V_2\rho_3c_3V_3}$$

$$P_{11} = \frac{K_{20}A_1h_1A_2h_2}{\rho_1c_1V_1\rho_2c_2V_2\rho_3c_3V_3}$$

APPENDIX B. TRANSFER FUNCTION FOR A SUPERHEATER TUBE WALL
DIVIDED INTO THREE LAYERS

From Eq. (2-D4)

$$\frac{d}{dt}\begin{bmatrix} \Delta T_1 \\ \Delta T_2 \\ \Delta T_3 \end{bmatrix} = \begin{bmatrix} \dfrac{-A_1 h_1}{\rho_1 c_1 V_1} & \dfrac{A_2 h_2}{\rho_1 c_1 V_1} & 0 \\ \dfrac{A_1 h_1}{\rho_2 c_2 V_2} & \dfrac{-(A_1 h_1 + A_2 h_2)}{\rho_2 c_2 V_2} & \dfrac{A_2 h_2}{\rho_2 c_2 V_2} \\ 0 & \dfrac{A_2 h_2}{\rho_3 c_3 V_3} & \dfrac{-A_3 h_3}{\rho_3 c_3 V_3} \end{bmatrix}\begin{bmatrix} \Delta T_1 \\ \Delta T_2 \\ \Delta T_3 \end{bmatrix}$$

$$+ \begin{bmatrix} \dfrac{1}{\rho_1 c_1 V_1} & 0 \\ 0 & 0 \\ 0 & \dfrac{-1}{\rho_3 c_3 V_3} \end{bmatrix}\begin{bmatrix} \Delta q_{gsh} \\ \Delta q_{Dsh} \end{bmatrix} \tag{B-1}$$

Equation (2-E5) can be written as

$$\Delta q_{Dsh} = K_{14}\Delta T_3 + K_{13}\Delta W_v + K_{15}\Delta D_D - K_{16}\Delta T_{sh} \tag{B-2}$$

Substitute Eq. (B-2) into Eq. (B-1)

$$\frac{d}{dt}\begin{bmatrix} \Delta T_1 \\ \Delta T_2 \\ \Delta T_3 \end{bmatrix} = \begin{bmatrix} \dfrac{-A_1 h_1}{\rho_1 c_1 V_1} & \dfrac{A_2 h_2}{\rho_1 c_1 V_1} & 0 \\ \dfrac{A_1 h_1}{\rho_2 c_2 V_2} & \dfrac{-(A_1 h_1 + A_2 h_2)}{\rho_2 c_2 V_2} & \dfrac{A_2 h_2}{\rho_2 c_2 V_2} \\ 0 & \dfrac{A_2 h_2}{\rho_3 c_3 V_3} & \dfrac{-(A_3 h_3 + K_{14})}{\rho_3 c_3 V_3} \end{bmatrix}\begin{bmatrix} \Delta T_1 \\ \Delta T_2 \\ \Delta T_3 \end{bmatrix}$$

$$+ \begin{bmatrix} \dfrac{1}{\rho_1 c_1 V_1} & 0 & 0 & 0 \\[2em] 0 & 0 & 0 & 0 \\[2em] 0 & \dfrac{-K_{13}}{\rho_3 c_3 V_3} & \dfrac{K_{15}}{\rho_3 c_3 V_3} & \dfrac{K_{16}}{\rho_3 c_3 V_3} \end{bmatrix} \begin{bmatrix} \Delta q_{sh} \\[1em] \Delta W_v \\[1em] \Delta D_D \\[1em] \Delta T_{sh} \end{bmatrix} \qquad (B-3)$$

where Δq_{sh}, ΔW_v, ΔD_D, ΔT_{sh} are the inputs and Eq. (B-2) is the output equation.

Following the same approach as in Appendix A, the following result is obtained,

$$\Delta T_3(s) = \frac{1}{s^4 + P_1 s^3 + P_2 s^2 + P_3 s + P_4} \left[P(s + P_6)\Delta q_{gsh}(s) \right.$$

$$\left. -(s^3 + P_7 s^2 + P_8 s + P_9)(P_{10}\Delta W_v(s) + P_{11}\Delta D_D(s) - P_{12}\Delta T_{sh}(s)) \right]$$

where

$$P_1 = \frac{A_3 h_3 + K_{14}}{\rho_3 c_3 V_3} + 2\frac{A_1 h_1 + A_2 h_2}{\rho_2 c_2 V_2} + \frac{A_1 h_1}{\rho_1 c_1 V_1}$$

$$P_2 = \frac{(A_3 h_3 + K_{14})(A_1 h_1 + A_2 h_2)}{\rho_2 c_2 V_2\,{}_3 c_3 V_3} + \frac{A_1 h_1}{\rho_1 c_1 V_1} + \frac{A_1 h_1 + A_2 h_2}{\rho_2 c_2 V_2} \times$$

$$\left(\frac{A_3 h_3 + K_{14}}{\rho_3 c_3 V_3} + \frac{A_1 h_1 + A_2 h_2}{\rho_2 c_2 V_2} \right) + \frac{A_1^2 h_1^2}{\rho_1 c_1 V_1 \rho_2 c_2 V_2} - \frac{A_2^2 h_2^2}{\rho_2 c_2 V_2 \rho_3 c_3 V_3}$$

$$P_3 = \frac{A_1^2 h_1^2 (A_1 h_1 + A_2 h_2)}{\rho_1 c_1 V_1 \rho_2 c_2 V_2 \rho_3 c_3 V_3} + \left(\frac{A_1 h_1}{\rho_1 c_1 V_1} + \frac{A_1 h_1 + A_2 h_2}{\rho_2 c_2 V_2} \right) \times$$

$$\frac{(A_3 h_3 + K_{14})(A_1 h_1 + A_2 h_2)}{\rho_2 c_2 V_2 \rho_3 c_3 V_3} + \frac{A_1^2 h_1^2 (A_1 h_1 + A_2 h_2)}{\rho_1 c_1 V_1 (\rho_2 c_3 V_2)^2}$$

$$- \frac{A_2^2 h_2^2}{\rho_2 c_2 V_2 \rho_3 c_3 V_3} \left(\frac{A_1 h_1 + A_2 h_2}{\rho_1 c_1 V_1 \rho_2 c_2 V_2} + \frac{A_1 h_1}{\rho_1 c_1 V_1} \right)$$

$$P_4 = \frac{A_2^2 h_2^2 (A_3 h_3 + K_{14})(A_1 h_1 + A_2 h_2)}{\rho_1 c_1 V_1 (\rho_2 c_2 V_2)^2 \rho_3 c_3 V_3} - \frac{A_1 h_1 (A_2 h_2)^2 (A_1 h_1 + A_2 h_2)}{(\rho_1 c_1 V_1 \rho_2 c_2 V_2)^2 \rho_3 c_3 V_3}$$

$$P_5 = \frac{A_1 h_1 A_2 h_2}{\rho_1 c_1 V_1 \rho_2 c_2 V_2 \rho_3 c_3 V_3}$$

$$P_6 = \frac{A_1 h_1 + A_2 h_2}{\rho_2 c_2 V_2}$$

$$P_7 = 2 \frac{A_1 h_1 + A_2 h_2}{\rho_2 c_2 V_2} + \frac{A_1 h_1}{\rho_1 c_1 V_1}$$

$$P_8 = \left(\frac{A_1 h_1}{\rho_1 c_1 V_1} + \frac{A_1 h_1 + A_2 h_2}{\rho_2 c_2 V_2} \right) \left(\frac{A_1 h_1 + A_2 h_2}{\rho_2 c_2 V_2} \right) + \frac{A_1^2 h_1^2}{\rho_1 c_1 V_1 \, 2 c_2 V_2}$$

$$P_9 = \frac{A_1^2 h_1^2 (A_1 h_1 + A_2 h_2)}{\rho_1 c_1 V_1 (\rho_2 c_2 V_2)^2}$$

$$P_{10} = \frac{\bar{K}_{13} A_1 h_1 A_2 h_2}{\rho_1 c_1 V_1 \rho_2 c_2 V_2 \rho_3 c_3 V_3}$$

$$P_{11} = \frac{K_{15} A_1 h_1 A_2 h_2}{\rho_1 c_1 V_1 \rho_2 c_2 V_2 \rho_3 c_3 V_3}$$

$$P_{12} = \frac{K_{16} A_1 h_1 A_2 h_2}{\rho_1 c_1 V_1 \rho_2 c_2 V_2 \rho_3 c_3 V_3}$$

APPENDIX C.   REDUCTION OF TRANSFER FUNCTION BLOCKS

To reduce the blocks
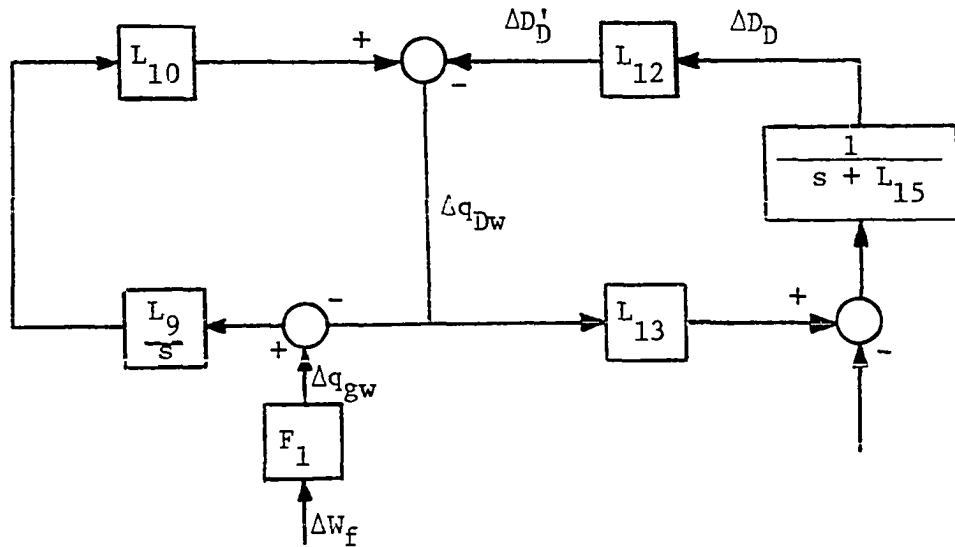
The left loop can be written as
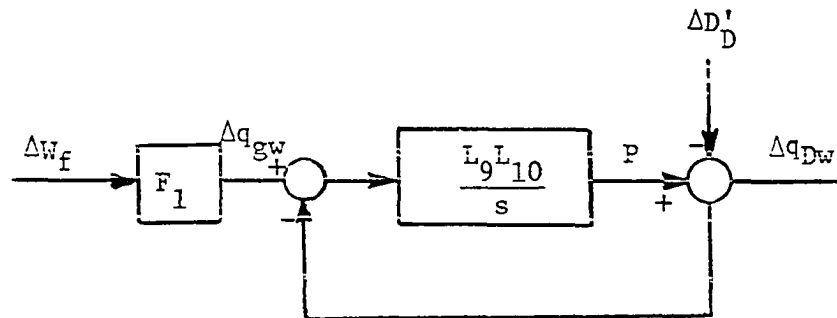
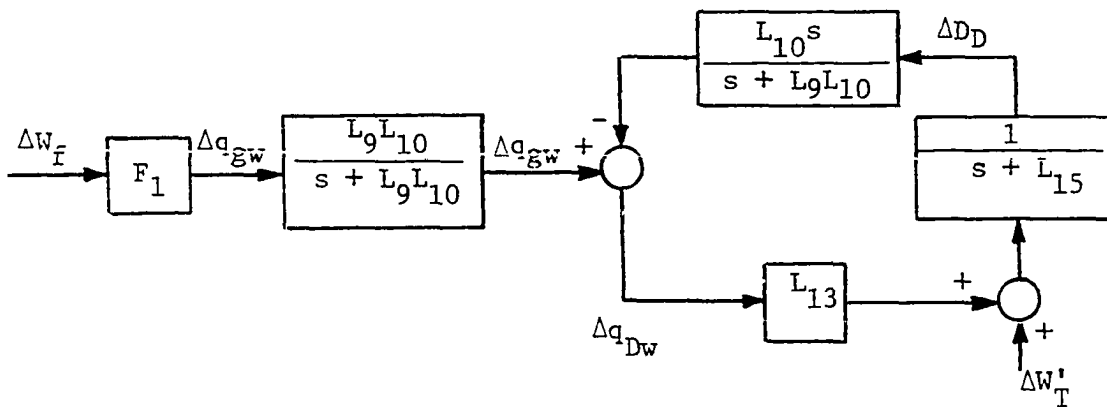$$\left\{\Delta q_{gw}(s) - [P(s) - \Delta D_D'(s)]\right\} \frac{L_9 L_{10}}{s} = P(s)$$

$$[\Delta q_{gw}(s) + \Delta D_D'(s)] \frac{L_9 L_{10}}{s} = \frac{s + L_9 L_{10}}{s} P(s)$$

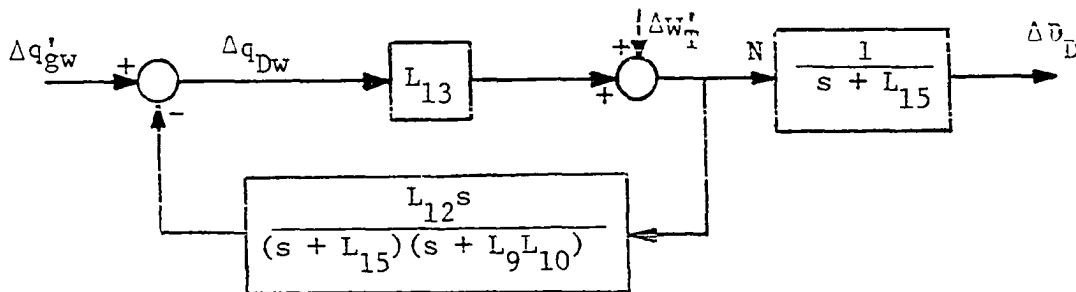$$[\Delta q_{gw}(s) + \Delta D_D'(s)] \frac{L_9 L_{10}}{s + L_9 L_{10}} = P(s)$$

Subtract $\Delta D_D'(s)$ from both sides,

$$\Delta q_{gw}(s) \frac{L_9 L_{10}}{s + L_9 L_{10}} - \Delta D_D'(s) \frac{s}{s + L_9 L_{10}} = \Delta P(s) - \Delta D_D'(s) = \Delta q_{Dw}(s)$$

the original blocks become

The loop on the right can be written as

$$\Delta q_{gw}'(s) - (L_{13} \Delta q_{Dw}(s) - \Delta W_T'(s)) \frac{L_{12} s}{(s + L_{15})(s + L_9 L_{10})} = \Delta q_{Dw}$$
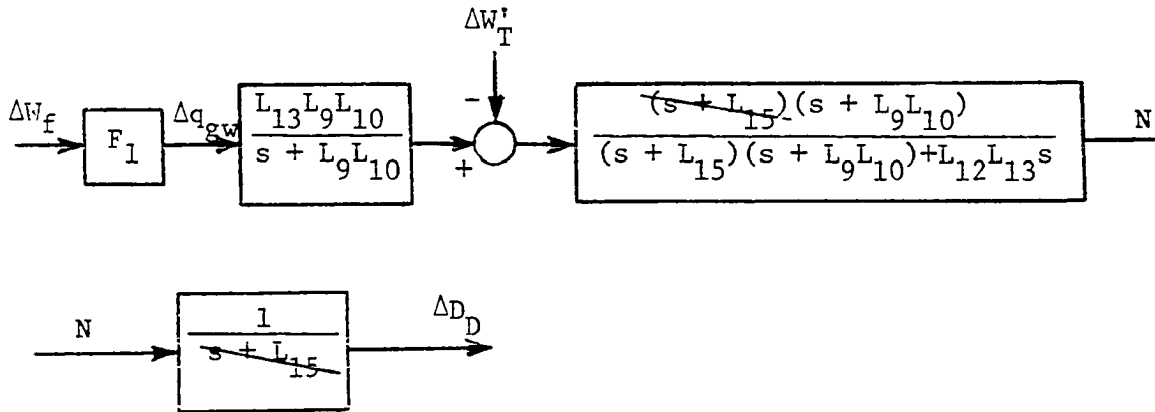
$$\Delta q_{gw}'(s) + \Delta W_T'(s) \frac{L_{12} s}{(s + L_{15})(s + L_9 L_{10})}$$

$$= \frac{L_{12}L_{13}s + (s + L_{15})(s + L_9L_{10})}{(s + L_{15})(s + L_9L_{10})} \Delta q_{Dw}(s)$$

$$\Delta q'_{gw}(s) \frac{(s + L_{15})(s + L_9L_{10})}{(s + L_{15})(s + L_9L_{10}) + L_{12}L_{13}s}$$

$$+ \Delta W'_T(s) \frac{L_{12}s}{(s + L_{15})(s + L_9L_{10}) + L_{12}L_{13}s} = \Delta q_{Dw}$$

Multiply both sides by L_{13}, then subtract by $\Delta W'_T$,

$$\Delta q'_{gw}(s) \frac{L_{13}(s + L_{15})(s + L_9L_{10})}{(s + L_{15})(s + L_9L_{10}) + L_{12}L_{13}s}$$

$$- \Delta W'_T(s) \frac{(s + L_{15})(s + L_9L_{10})}{(s + L_{15})(s + L_9L_{10}) + L_{12}L_{13}s} = L_{13}\Delta q_{Dw} - \Delta W'_T = N$$

Then the block becomes

Let

$$L_9L_{10} = P_1$$

$$F_1L_{13}L_9L_{10} = P_2$$

$$L_{15} + L_9L_{10} + L_{12}L_{13} = P_3$$

$$L_{15}L_9L_{10} = P_4$$

The original block becomes